MACROECONOMIC GOALS AND INFLATION TARGETING IN INDIA

A THESIS

SUBMITTED TO



UNIVERSITY OF KOTA, KOTA

BY GIRISH KUMAR PALIWAL

UNDER THE SUPERVISION OF DR. G. L. MALAV LECTURER, GOVERNMENT COLLEGE, KOTA

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF DEGREE OF

DOCTOR OF PHILOSOPHY IN ECONOMICS

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holiest thing I have ever seen alive, my mother, the dearest one on the earth.

I, myself, typed and formatted this thesis. All errors and inconsistencies

are my own.

Date: 25th April, 2013, Shri Hanuman Jayanti

Place: Kota

Girish Kumar Paliwal

[i]

Declaration

I, Girish Kumar Paliwal, do hereby, declare that the research work carried

out in the thesis entitled "Macroeconomic Goals and Inflation Targeting in India"

submitted to the University of Kota, Kota for fulfillment of partial requirements

for the award of degree of Doctor of Philosophy in Economics is a record of my

bona-fide research work except mentioned otherwise and has not been

previously submitted in any university or institute for award of any degree,

diploma or fellowship.

Date: 25th April, 2013, Shri Hanuman Jayanti

Place: **Kota**

Girish Kumar Paliwal

[ii]

Certificate

I feel immense pleasure to certify that the thesis entitled "Macroeconomic

Goals and Inflation Targeting in India" is a record of bona-fide research work of

Mr. Girish Kumar Paliwal except mentioned otherwise has been carried out

under my supervision.

I find that research work based on pure theory carried out by Mr. Paliwal

is of high quality. In my opinion aforementioned thesis is fully adequate in scope

and quality as a PhD thesis and well worth to submit. I recommend it for

submission to the University of Kota, Kota for fulfillment of partial requirements

for the award of degree of Doctor of Philosophy in Economics.

To the best of my knowledge no part of the aforesaid thesis has previously

been submitted in any university or institute for award of any degree, diploma or

fellowship.

Mr. Paliwal has completed the residential requirement by residing (and

appearing before me at Government College, Kota) at the headquarters of my

workplace i.e. Kota as per the rules of the University of Kota, therefore,

residential obligation as per rules has been satisfied by Mr. Paliwal.

Date: 13th May, 2013

Place: Kota

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A Word on Thesis

This Thesis comprises of two short essays, namely, "New Keynesian

Model for Indian Economy" and "Inflation Targeting Model for Indian Economy".

Both of them (essays) together build the Thesis, "Macroeconomic Goals and

Inflation Targeting in India".

Date: 25th April, 2013, Shri Hanuman Jayanti

Place: Kota

Girish Kumar Paliwal

[vi]

I New Keynesian Model for Indian Economy

Abstract

In this study I develop a New Keynesian Model to outfit the Indian economy and thereby to explain the nature of Indian domestic inflation, a key instrument for inflation targeting central banks. The Indian economy is an emerging market economy and primarily comprises of two sectors, namely, formal and informal and they are asymmetric in nature to each other. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector characterizes the complete flexibility in prices and wages and perfections in markets. Thus, Indian economy comprises of a very typical mixture of Keynesian and Classical markets. The study shows that when Reserve Bank of India conducts monetary policy in such an environment then nominal and real effects (in short run) are observed in informal and formal sector markets, respectively. The main interest of this study is to study the nature of Indian domestic inflation and thereby to study the real variables of the economy. The New Keynesian Phillips Curve reveals that the degree of stickiness in prices in formal sector markets has a deep impact on the domestic inflation as informal sector markets are frictionless and have complete price flexibility (zero stickiness). Thus, degree of stickiness in prices in formal sector markets plays a major role to determine the domestic inflation and enables the monetary policy to stabilize formal sector output. Compactly, monetary policy affects the real variables of the economy in formal sector in short run while nominal variables (price and wage level) in informal sector. Thus, the study reveals that monetary policy in India has a very poor control on real variables of the economy in short run due to presence of huge informal sector.

IEL Classification: E12, E31, E32, E50, E51, E52, E58, E63, F41

Keywords: New Keynesian Model, Informal Sector, Sticky Prices,

Domestic Inflation, New Keynesian Phillips Curve, Dynamic

IS Curve, Taylor Rule, India

I believe myself to be writing a book on economic theory which will largely revolutionize – not I suppose, at once but in the course of next ten years¹.

John Maynard Keynes

1 Resurgence of Keynesianism²

The New Classical School of thought comprises of earthshaking talents like Thomas Sargent, Neil Wallace, Edward Prescott and Robert Barro led by Robert Lucas, has exposed the very serious theoretical flaws and inconsistencies in the orthodox Keynesian theory in 1970s. The outclass orthodox Keynesian model, the only game in the town till 1960s in terms of macroeconomic policy, was totally failed to pass the empirical test of 1970s as the vital pillar of orthodox Keynesianism, the Phillips Curve, collapsed. A number of seminal papers in a row of Robert Lucas in 1970s pushed the celebrated Keynesian policy regime in a corner under pitiable conditions. That was a humiliated defeat of Keynesians.

The attack of Sweetwater Economists³ was not left unanswered. A mighty gang of superb people like Gregory Mankiw and Lawrence Summers (Harvard); Olivier Blanchard (IMF and at MIT), Stanley Fischer (Bank of Israel, formally at World Bank and at MIT); Michael Woodford (Columbia, formally at Princeton), Bruce Greenwald, Edmund Phelps and Joseph Stiglitz (Columbia); Jordi Gali (Pompeu Fabra, formally at New York and at Columbia); Ben Bernanke (Federal Reserve System and formally at Princeton); Laurence Ball (Johns Hopkins); George Akerlof, Janet Yellen and David Romer (Berkeley); Robert Hall and John Taylor (Stanford); Dennis Snower (Kiel) and Assar Lindbeck (Stockholm) who are the best in their profession on the green planet counterattacked on the ideas of New Classical School of thought and reestablished the Keynesian magic. This school of thought is labeled as New Keynesian Economics and a new term New Keynesian⁴ was emerged in the literature. Though, the New Keynesian Models are dissimilar in many aspects with their distant cousins of 1960s but still hold

1

The letter of John Maynard Keynes reads written on New Year's Day, 1935 to his friend George Bernard Shaw for his forthcoming masterpiece "The General Theory".

Snowdon and Vane's book "Modern Macroeconomics: Its Origins, Development and Current State", published in 2005 has dominated this section.

³ Sweetwater Economists are referred as New Classical Economists.

⁴ Parkein and Bade in their book "Modern Macroeconomics" firstly used the term New Keynesian published by Oxford: Philip Allan in 1982.

the main ideas of Keynesian revolution⁵. Summarily, the New Keynesian Economics is school of thought in modern Macroeconomics that evolved from the ideas of the father of modern Macroeconomics John Maynard Keynes in response to the New Classical School of thought.

The Saltwater Economists⁶ rigorously and robustly explain the non-neutrality of money and market imperfections. Sticky prices and wages make money non-neutral and a market imperfection explains this behavior of prices and wages. Thus, non-neutrality of money (sticky prices and wages) and market imperfections (absence of continuous market clearing) are the hallmarks of New Keynesian theory.

Over the years Rotemberg and Woodford (1995, 1997), Yun (1996), Good-friend and King (1997), Clarida, Gali and Gertler (1999), McCallum and Nelson (1999), McCallum and Nelson (2000), Clarida, Gali and Gertler (2001), Gali (2002), Woodford (2003), Benigno and Benigno (2003, 2008), Benigno (2004), Gali and Monacelli (2005) and Gali (2008) are few among the countless others who have refined the New Keynesian Models. Since beginning of this millennium New Keynesian Models have won vote of confidence and have become the workhorse models to analyze the business cycle and monetary policy. New Keynesian Phillips Curve, Dynamic IS Curve and monetary policy rule (Taylor rule) are the key building blocks of the New Keynesian Models.

In this study I develop a New Keynesian Model to outfit the Indian economy and thereby to target the domestic price inflation, a key instrument for inflation targeting central banks. The Indian economy is an emerging market economy and it has all the characteristics of an emerging market economy. Indian economy, primarily, comprises of two sectors, namely, formal and informal and they are asymmetric in nature to each other. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector of Indian economy characterizes the complete flexibility in prices and wages and perfections in markets. Thus, Indian economy comprises of Keynesian markets in the formal sector and Classical markets in the informal

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⁵ "The General Theory of Employment, Interest and Money" a world shattering book written by John Maynard Keynes right after the great depression which published in February, 1936 is referred as the Keynesian revolution.

⁶ Saltwater Economists are referred as New Keynesian Economists.

sector. To represent New Keynesian and New Classical ideas in a single model is a herculean task but I model them together in a single model to represent the behavior of Indian economy. Handling the typical mixture of sticky and flexible prices is a crucial property of my model.

2 Informality in Indian Economy

The structure of emerging market economies is somewhat differ than that of advance economies due to existence of large informal sector. The structure of goods, labour and credit markets are pretty dissimilar in formal and informal sectors of the economy as agents have different endowments and constraints. In the advance economies the relative size of informal sector is much smaller to that of formal sector; therefore, it is reasonable to ignore the informal sector in advanced economies as it has negligible impact on the aggregates. But in the emerging market economies where the informal sector is relatively large and plays an important role in the economy then neglecting the informal sector would not be justified; Schneider et al. (2010). Informal sector plays a major role in employment generation, especially for the developing world; Agenor and Montiel (1996); Harris-White and Sinha (2007); Marjit and Kar (2011) and Dutta et al. (2011). The informal sector is always complex to deal with as most of the activities of this sector are gone unrecorded.

Unorganised or informal sector constitutes a pivotal part of the Indian economy. More than 90 per cent of workforce and about 50 per cent of the national product are accounted for by the informal economy. A high proportion of socially and economically underprivileged sections of society are concentrated in the informal economic activities. The high level of growth of the Indian economy during the past two decades is accompanied by increasing informalisation. There are indications of growing interlinkages between informal and formal economic activities. There has been new dynamism of the informal economy in terms of output, employment and earnings. Faster and inclusive growth needs special attention to informal economy. Sustaining high levels of growth are also intertwined with improving domestic demand of those engaged

in informal economy, and addressing the needs of the sector in terms of credit, skills, technology, marketing and infrastructure, NSC⁷ (2012).

A number of studies have been conducted to trace out the effects of informality on the economy. Some of them are as under:

Batini et al. (2010) explore the costs and benefits of informality associated with the informal sector lying outside the tax regime in a two-sector New Keynesian model. The informal sector is more labour intensive, has a lower labour productivity, is untaxed and has a classical labour market. The formal sector bears all the taxation costs, produces all the government services and capital goods, and wages are determined by a real wage norm.

Batini et al. (2011) construct a two-sector, formal-informal new Keynesian closed-economy model. The informal sector is more labour intensive, is untaxed, has a classical labour market, faces high credit constraints in financing investment and is less visible in terms of observed output.

Bridji and Charpe (2011) develop a model of an economy with dual labour markets to understand the dynamics of the informal sector over the business cycle, as well as to analyze the implication of duality for the volatility of output and the persistence of employment. The informal labour market is competitive while the formal labour market is characterized by search costs. The size of each labour market segment depends on labour demand by firms as well as participation decisions of households. Authors show that the informal sector plays the role of a buffer, expanding in periods of recessions and shrinking when recovery sets in. Authors also show that workers switching between the two labour market increases the volatility of output. Finally, labour market segmentation modifies the properties of the search model, as the competitive labour market segment reduces the volatility of employment, unless transition costs are high.

Castillo and Montoro (2009) analyze the effects of informal labor markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks. In doing so, authors incorporate the informal sector in a modified New Keynesian model with labor market frictions as in the Diamond-Mortensen-

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National Statistical Commission, Government of India, New Delhi.

Pissarides model. Authors show that the informal economy generates a "buffer" effects that diminish the pressure of demand shocks on aggregate wages and inflation.

Gabriel et al. (2011) develop a closed-economy DSGE model of the Indian economy and estimate it by Bayesian Maximum Likelihood methods using Dynare. Authors build up in stages to a model with a number of features important for emerging economies in general and the Indian economy in particular: a large proportion of credit-constrained consumers, a financial accelerator facing domestic firms seeking to finance their investment and an informal sector.

Goyal (2007) represents an optimizing model of a small open emerging market economy (SOEME) with dualistic labour markets and two types of consumers, delivers a tractable model for monetary policy.

Goyal (2008) develops a simplified version of a typical dynamic stochastic open economy general equilibrium models used to analyze optimal monetary policy. Author outlines the chief modifications when dualism in labour and in consumption is introduced to adapt the model to a small open emerging market such as India. The implications of specific labour markets, and the structure of Indian inflation and its measurement are examined.

Haider et al. (2012) develop an open economy dynamic stochastic general equilibrium (DSGE) model based on New-Keynesian micro-foundations. Alongside standard features of emerging economies, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports. Authors also incorporate informal labor and production sectors. This customization intensifies the exposure of a developing economy to internal and external shocks in a manner consistent with the stylized facts of Business Cycle Fluctuations.

3 Households

The Indian economy has relatively very large informal sector as the lion's share of Indian workforce works in this sector to contribute around half of its national product. In such an informal economic environment this study studies

the nature of domestic inflation and thereby studies the real variables of the economy i.e. output and employment. The related issues have been framed in an Open Economy New Keynesian Dynamic Stochastic General Equilibrium Model with micro-foundations.

The world economy is modeled⁸ as a continuum of small open economies with identical preferences, technology, and market structure, indexed by a unit interval[0,1], so as it does not have any impact of policy decisions of any economy as in Gali and Monacelli (2005). Again, the home economy is divided in to two sectors, namely, formal and informal following Conesa, et al. (2002); Ihrig and Moe (2004); Batini et al. (2010) and Batini et al. (2011). Each sector of home economy is populated by continuum of households and spreads on a unit mass[0,1] with population size $\gamma: (1-\gamma):: formal\ sector: informal\ sector,$ moreover, each of the sectors consumes/produces continuum of differentiated goods as her population size.

The home economy is inhabited by an infinitely lived representative household who derives its utility from additively separable utility function comprises of consumption and leisure (negative utility from working/production) as $U(C_t, L_t)$ and wishes to maximize the utility following Walsh (2003) and Woodford (2003) as:

$$Max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

The CRRA⁹ (period) utility is given by as in Gali (2008):

$$Max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu} \right\}$$
 (1.1.1)

Where

⁸ Government spending (consumption spending and capital spending), money and habit formation are excluded from the model to keep it simple.

Constant Relative Risk Aversion Utility.

$$U(C_t, L_t) = \left(\frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu}\right)$$

Subject to

$$C_t P_t + E_t \{Q_{t,t+1} B_{t+1}\} = B_t + L_t W_t - T_t$$
 (1.1.2)

Where C_t, L_t are consumption and labour supply indices, respectively. $U(\cdot)$ is utility function. ε and v are intertemporal substitutability (elasticities of substitution) of consumption and that of labour supply between periods. β is discount factor, E_0 is expectational operator, P_t, W_t are general price level and nominal general wage level, T_t government transfer minus distorted tax, B_t represents the quantity of one period, nominally riskless discounted bonds purchased in the period t and maturing in the period t+1. B_{t+1} is nominal payoff in the period t+1 of the portfolio held at the end of the period t. Each bond pay one unit of money at maturity and its price is Q_t and $Q_t = \frac{1}{1+i_t}$, where, i_t is nominal interest rate. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead nominal payoffs relevant to domestic household, moreover, it is assumed as in Gali (2008) that households have access to a complete set of contingent claims, traded internationally.

Economy wide total expenditure¹⁰ $C_t P_t$ of the domestic households on the consumption and economy wide total nominal wage income¹¹ $L_t W_t$ of the domestic households can be given as:

$$C_{t}P_{t} = \int_{0}^{1} [C_{HF,t}(i)][P_{HF,t}(i)]di$$

$$+ \int_{0}^{1} [C_{HI,t}(i)][P_{HI,t}(i)]di + \int_{0}^{1} \int_{0}^{1} [C_{j,t}(i)][P_{j,t}(i)]didj$$
(1.1.3)

¹⁰ (A.1.1) to (A.1.5) in Appendix A make (1.1.3).

⁽A.1.7) to (A.1.9) in Appendix A make (1.1.4).

$$L_t W_t = \int_0^1 [L_{HF,t}(i)] [W_{HF,t}(i)] di + \int_0^1 [L_{HI,t}(i)] [W_{HI,t}(i)] di$$
 (1.1.4)

The domestic composite consumption aggregator C_t can be given following Dixit and Stiglitz (1977) as:

$$C_{t} = \left[(1 - \alpha)^{\frac{1}{\vartheta_{a}}} \left(C_{H,t} \right)^{\frac{\vartheta_{a} - 1}{\vartheta_{a}}} + (\alpha)^{\frac{1}{\vartheta_{a}}} \left(C_{F,t} \right)^{\frac{\vartheta_{a} - 1}{\vartheta_{a}}} \right]^{\frac{\vartheta_{a} - 1}{\vartheta_{a} - 1}}$$
(1.1.5)

Where $C_{H,t}$ and $C_{F,t}$ are indices of domestic consumption of domestically and foreign produced goods, respectively and ϑ_a is intratemporal substitutability (elasticity of substitution) of consumption between domestically and foreign produced goods. α is degree of openness while $1-\alpha$ is home biasness. The analogous CES aggregator of domestically produced goods $C_{H,t}$ can be given as:

$$C_{H,t} = \left[(\gamma)^{\frac{1}{\vartheta_b}} \left(C_{HF,t} \right)^{\frac{\vartheta_b - 1}{\vartheta_b}} + (1 - \gamma)^{\frac{1}{\vartheta_b}} \left(C_{HI,t} \right)^{\frac{\vartheta_b - 1}{\vartheta_b}} \right]^{\frac{\vartheta_b}{\vartheta_b - 1}}$$
(1.1.6)

Where $C_{HF,t}$ and $C_{HI,t}$ are indices of domestic consumption of domestic formal and domestic informal sectors produced goods, respectively and ϑ_b is intratemporal elasticity of substitution of consumption between formal and informal sector produced goods. γ and $1-\gamma$ are share of domestic consumption of formal and informal sector produced goods, respectively. The CES function of domestic consumption of domestic formal sector produced goods $C_{HF,t}$ can be given as following Woodford (2003):

$$C_{HF,t} = \left[\int_{0}^{1} \left[C_{HF,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(1.1.7)

Where $[C_{HF,t}(i)]$ is the quantity of good i produced in the domestic formal sector and domestically consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of domestic formal sector produced goods. The CES function of domestic consumption of domestic informal sector produced goods $C_{HI,t}$ can be given as:

$$C_{HI,t} = \left[\int_{0}^{1} \left[C_{HI,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(1.1.8)

Where $[C_{HI,t}(i)]$ is the quantity of good i produced in the domestic informal sector and domestically consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of informal sector produced goods. The CES function of domestic consumption of foreign produced goods $C_{F,t}$ can be given as:

$$C_{F,t} = \left[\int_{0}^{1} \left[C_{j,t} \right]^{\frac{\vartheta_{d}-1}{\vartheta_{d}}} dj \right]^{\frac{\vartheta_{d}}{\vartheta_{d}-1}}$$
(1.1.9)

Where $C_{j,t}$ is the index of domestic consumption of country j produced goods and ϑ_d is intratemporal elasticity of substitution of consumption of goods produced in different countries of the world. The CES function of domestic consumption of country j produced goods $C_{j,t}$ can be given as:

$$C_{j,t} = \left[\int_{0}^{1} \left[C_{j,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(1.1.10)

Where $[C_{j,t}(i)]$ is the quantity of good i produced in country j and domestically consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of country j produced goods. The corresponding

consumption based price indices of (1.1.5) to (1.1.10) are given by (1.1.11) to (1.1.16), respectively as under¹² following Benigno and Benigno (2003) and Benigno (2004):

$$P_{t} = \left[(1 - \alpha) (P_{H,t})^{1 - \vartheta_{a}} + (\alpha) (P_{F,t})^{1 - \vartheta_{a}} \right]^{\frac{1}{1 - \vartheta_{a}}}$$
(1.1.11)

$$P_{H,t} = \left[\gamma (P_{HF,t})^{1-\vartheta_b} + (1-\gamma) (P_{HI,t})^{1-\vartheta_b} \right]^{\frac{1}{1-\vartheta_b}}$$
 (1.1.12)

$$P_{HF,t} = \left[\int_{0}^{1} \left[P_{HF,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
 (1.1.13)

$$P_{HI,t} = \left[\int_{0}^{1} \left[P_{HI,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
(1.1.14)

$$P_{F,t} = \left[\int_{0}^{1} [P_{j,t}]^{1-\vartheta_d} dj \right]^{\frac{1}{1-\vartheta_d}}$$
 (1.1.15)

$$P_{j,t} = \left[\int_{0}^{1} \left[P_{j,t}(i) \right]^{1-\vartheta_c} di \right]^{\frac{1}{1-\vartheta_c}}$$
 (1.1.16)

Optimal allocation of goods derives the following demand functions in each category for given level of expenditure as^{13} :

$$C_{H,t} = (1 - \alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} C_t$$
 (1.1.17)

¹² (A.1.12) in the Appendix A derives (1.1.13). (1.1.11) to (1.1.12) and (1.1.14) to (1.1.16) can, analogously, be derived.

¹³ (A.1.11) in Appendix A derives (1.1.21). (1.1.17) to (1.1.20) and (1.1.22) to (1.1.24) can, analogously, be derived.

$$C_{F,t} = (\alpha) \left(\frac{P_{F,t}}{P_t}\right)^{-\vartheta_a} C_t$$
 (1.1.18)

$$C_{HF,t} = \gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} C_{H,t}$$
 (1.1.19)

$$C_{HI,t} = (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b} C_{H,t}$$
 (1.1.20)

$$\left[C_{HF,t}(i)\right] = \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\vartheta_c} C_{HF,t}$$
(1.1.21)

$$\left[C_{HI,t}(i)\right] = \left(\frac{\left[P_{HI,t}(i)\right]}{P_{HI,t}}\right)^{-\vartheta_c} C_{HI,t}$$
(1.1.22)

$$\left[C_{j,t}(i)\right] = \left(\frac{\left[P_{j,t}(i)\right]}{P_{j,t}}\right)^{-\vartheta_c} C_{j,t}$$
(1.1.23)

$$C_{j,t} = \left(\frac{P_{j,t}}{P_{F,t}}\right)^{-\vartheta_d} C_{F,t}$$
(1.1.24)

The domestic labour supply aggregator L_t , analogous to **(1.1.5)** can be given following Dixit and Stiglitz (1977) as:

$$L_{t} = \left[(\gamma)^{\frac{1}{\vartheta_{e}}} (L_{HF,t})^{\frac{\vartheta_{e}-1}{\vartheta_{e}}} + (1-\gamma)^{\frac{1}{\vartheta_{e}}} (L_{HI,t})^{\frac{\vartheta_{e}-1}{\vartheta_{e}}} \right]^{\frac{\vartheta_{e}}{\vartheta_{e}-1}}$$
(1.1.25)

Where $L_{HF,t}$ and $L_{HI,t}$ are indices of domestic labour supply in domestic formal and informal sectors, respectively and ϑ_e is intratemporal elasticity of substitution of labour supply between formal and informal sectors. γ and $1 - \gamma$ are share of domestic labour supply in formal and informal sectors, respectively.

The CES function of labour supply in domestic formal sector $L_{HF,t}$ can be given as under:

$$L_{HF,t} = \left[\int_{0}^{1} \left[L_{HF,t}(i) \right]^{\frac{\vartheta_f - 1}{\vartheta_f}} di \right]^{\frac{\vartheta_f}{\vartheta_f - 1}}$$
(1.1.26)

Where $\left[L_{HF,t}(i)\right]$ is the quantity of type i labour supplied in domestic formal sector in period t and ϑ_f is intratemporal elasticity of substitution between varieties of labour supplied to formal sector. The CES function of labour supply in domestic informal sector $L_{HI,t}$ can be given as:

$$L_{HI,t} = \left[\int_{0}^{1} \left[L_{HI,t}(i) \right]^{\frac{\vartheta_{f}-1}{\vartheta_{f}}} di \right]^{\frac{\vartheta_{f}}{\vartheta_{f}-1}}$$
(1.1.27)

Where $[L_{HI,t}(i)]$ is the quantity of type i labour supplied in domestic informal sector in period t and ϑ_f is intratemporal elasticity of substitution between varieties of labour supplied to informal sector. The corresponding labour supply based wage indices of (1.1.25) to (1.1.27) are given by (1.1.28) to (1.1.30), respectively as under¹⁴:

$$W_{t} = \left[\gamma (W_{HF,t})^{1-\vartheta_{e}} + (1-\gamma) (W_{HI,t})^{1-\vartheta_{e}} \right]^{\frac{1}{1-\vartheta_{e}}}$$
 (1.1.28)

$$W_{HF,t} = \left[\int_{0}^{1} \left[W_{HF,t}(i) \right]^{1-\vartheta_f} di \right]^{\frac{1}{1-\vartheta_f}}$$
 (1.1.29)

Wage indices can, analogously, be derived as prices indices.

$$W_{HI,t} = \left[\int_{0}^{1} \left[W_{HI,t}(i) \right]^{1-\vartheta_f} di \right]^{\frac{1}{1-\vartheta_f}}$$
 (1.1.30)

Optimal allocation of labour derives the following supply functions in each category for given level of wage income.

$$\left[L_{HF,t}(i)\right] = \left(\frac{\left[W_{HF,t}(i)\right]}{W_{HF,t}}\right)^{-\vartheta_f} L_{HF,t} \tag{1.1.31}$$

$$\left[L_{HI,t}(i)\right] = \left(\frac{\left[W_{HI,t}(i)\right]}{W_{HI,t}}\right)^{-\vartheta_f} L_{HI,t}$$
(1.1.32)

$$L_{HF,t} = \gamma \left(\frac{W_{HF,t}}{W_t}\right)^{-\vartheta_f} L_t \tag{1.1.33}$$

$$L_{HI,t} = (1 - \gamma) \left(\frac{W_{HI,t}}{W_t}\right)^{-\vartheta_f} L_t$$
 (1.1.34)

3.1 Optimal Preferences of Households

(1.1.1) and (1.1.2) write the optimal consumption-saving decision 15 (optimal inter-temporal consumption decision, the consumption Euler equation) and the optimal consumption-leisure decision 16 (optimal consumption-labour supply decision), respectively, as:

$$1 = E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\} \frac{\beta}{Q_t}$$
 (1.1.35)

$$\frac{W_t}{P_t} = C_t^{\ \varepsilon} L_t^{\ \nu} \tag{1.1.36}$$

¹⁵ (A.1.20) in the Appendix A makes (1.1.35).

⁽A.1.21) in the Appendix A makes (1.1.36).

The log-linearization of (1.1.35) and (1.1.36) can be given by $(1.1.37)^{17}$ and $(1.1.38)^{18}$, respectively, as:

$$c_t = E_t c_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1})$$
 (1.1.37)

$$\varepsilon c_t + \nu l_t = w_t - p_t \tag{1.1.38}$$

Where small letter is the logarithm (with natural base) value of her corresponding capital letter and hereinafter the very same methodology is used throughout the text.

4 International Economic Environment

4.1 Terms of Trade

Bilateral terms of trade between domestic economy and country j is defined as price of country j's goods in terms of home goods:

$$S_{j,t} \equiv \frac{P_{j,t}}{P_{H,t}}$$
 (1.1.39)

The effective terms of trade are thus given by:

$$S_t = \frac{P_{F,t}}{P_{H,t}} {(1.1.40)}$$

$$S_t = \left(\int_0^1 \left[S_{j,t}\right]^{1-\vartheta_d} dj\right)^{\frac{1}{1-\vartheta_d}}$$

¹⁷ (A.1.24) in the Appendix A makes (1.1.37).

¹⁸ (A.1.25) in the Appendix A makes (1.1.38).

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 \left[S_{j,t}\right]^{1-\vartheta_d} dj\right)^{\frac{1}{1-\vartheta_d}}$$

Log linearization makes:

$$s_t = \int_0^1 s_{j,t} \, dj = p_{F,t} - p_{H,t}$$
 (1.1.41)

4.2 The CPI Inflation

CPI index is given by (1.1.11) as:

$$P_t = \left[(1 - \alpha) \left(P_{H,t} \right)^{1 - \vartheta_a} + (\alpha) \left(P_{F,t} \right)^{1 - \vartheta_a} \right]^{\frac{1}{1 - \vartheta_a}}$$

Log linearization makes:

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t}$$
 (1.1.42)

(1.1.41) and (1.1.42) make:

$$p_t = (1 - \alpha)p_{H,t} + \alpha(p_{H,t} + s_t)$$

$$p_t = p_{H,t} - \alpha p_{H,t} + \alpha p_{H,t} + \alpha s_t$$

$$p_t = p_{H,t} + \alpha s_t \tag{1.1.43}$$

CPI inflation is given by:

$$\pi_t = p_{t+1} - p_t$$

Plugging (1.1.43)

$$\pi_t = p_{H,t+1} + \alpha s_{t+1} - \left(p_{H,t} + \alpha s_t \right)$$

$$\pi_t = p_{H,t+1} - p_{H,t} + \alpha s_{t+1} - \alpha s_t$$

4.3 Domestic inflation

Domestic inflation is given by:

$$\pi_{H,t} = p_{H,t+1} - p_{H,t} \tag{1.1.44}$$

Plugging **(1.1.44)** and $\Delta s_t = s_{t+1} - s_t$

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \tag{1.1.45}$$

Where π_t and $\pi_{H,t}$, respectively, represent CPI and domestic inflation.

4.4 Real exchange rate

Assume that the law of one price holds for individual goods at all times (both for import and export prices), implying that:

$$P_{i,t}(i) = \epsilon_{i,t} [P_{i,t}^{j}(i)] \forall i,j \in [0,1]$$
 (1.1.46)

Where $\epsilon_{j,t}$ bilateral nominal exchange rate is defined as prices of country j currency in terms of domestic currency. $[P_{j,t}^j(i)]$ is the price of country j's good i expressed in terms of own currency i.e. in the currency of country j itself.

Plugging (1.1.46) in (1.1.16)

$$P_{j,t} = \left[\int_{0}^{1} \left(\epsilon_{j,t} [P_{j,t}^{j}(i)] \right)^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$

$$P_{j,t} = \left(\epsilon_{j,t}\right)^{\frac{1-\vartheta_c}{1-\vartheta_c}} \left[\int_0^1 \left(\left[P_{j,t}^j(i) \right] \right)^{1-\vartheta_c} di \right]^{\frac{1}{1-\vartheta_c}}$$

$$P_{j,t} = \epsilon_{j,t} \left[\int_{0}^{1} \left(\left[P_{j,t}^{j}(i) \right] \right)^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$

$$P_{j,t} = \epsilon_{j,t} P_{j,t}^j \tag{1.1.47}$$

Where $P_{j,t}^{j}$ is domestic prices index of country j and can be given, analogous to **(1.1.16)**, as in her respective domestic prices index:

$$P_{j,t}^{j} = \left[\int_{0}^{1} \left[P_{j,t}^{j}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$

Inserting (1.1.47) in (1.1.15) to make:

$$P_{F,t} = \left[\int_{0}^{1} \left[\epsilon_{j,t} P_{j,t}^{j} \right]^{1-\vartheta_d} dj \right]^{\frac{1}{1-\vartheta_d}}$$

Log linearization makes:

$$p_{F,t} = \int_{0}^{1} (e_{j,t} + p_{j,t}^{j}) dj$$

$$p_{F,t} = \int_{0}^{1} e_{j,t} \, dj + \int_{0}^{1} p_{j,t}^{j} \, dj$$

$$p_{F,t} = e_t + p_t^W {(1.1.48)}$$

Where e_t is effective nominal exchange rate and p_t^W is world price level and can be given as:

$$e_t = \int_0^1 e_{j,t} \, dj \tag{1.1.49}$$

$$p_t^W = \int_0^1 p_{j,t}^j \, dj$$
 (1.1.50)

Plugging (1.1.48) in (1.1.41)

$$s_t = e_t + p_t^W - p_{H,t} (1.1.51)$$

For the world as a whole there is neither distinction between domestic and CPI price level nor between their corresponding inflation rates. $p_{j,t}^j$ and p_t^j are the domestic and CPI price indices of the generic country j.

$$p_t^W = \int_0^1 p_t^j \, dj \tag{1.1.52}$$

The bilateral real exchange rate between home and country j is defined as the ratio of two countries CPI and both are express in terms of domestic currency.

$$Q_{j,t} = \frac{\epsilon_{j,t} P_t^j}{P_t} \tag{1.1.53}$$

Log linearization:

$$\log Q_{j,t} = \log \frac{\epsilon_{j,t} P_t^j}{P_t}$$

$$\log Q_{j,t} = \log \epsilon_{j,t} + \log P_t^j - \log P_t$$

$$q_{j,t} = e_{j,t} + p_t^j - p_t (1.1.54)$$

$$q_t = \int_0^1 q_{j,t} \, dj \tag{1.1.55}$$

Plugging (1.1.54) in (1.1.55)

$$q_{t} = \int_{0}^{1} \left(e_{j,t} + p_{t}^{j} - p_{t} \right) dj$$

$$q_t = \int_0^1 e_{j,t} \, dj + \int_0^1 p_t^j \, dj - \int_0^1 p_t \, dj$$

$$p_t = \int_0^1 p_t \, dj \tag{1.1.56}$$

Plugging (1.1.49), (1.1.52) and (1.1.56) make:

$$q_t = e_t + p_t^W - p_t$$

Plugging (1.1.51)

$$q_t = s_t + p_{H,t} - p_t$$

Plugging (1.1.43)

$$q_t = s_t + p_{H,t} - p_{H,t} - \alpha s_t$$

$$q_t = s_t (1 - \alpha) {(1.1.57)}$$

Thus, (1.1.57), relates terms of trade to real exchange rate.

4.5 International risk sharing

Under the assumption of complete international financial markets and perfect capital mobility, the expected nominal return from risk free bonds, in terms of domestic currency, must be the same as the expected domestic currency return from foreign bonds. With this relationship, we can equate the intertemporal optimality conditions for the domestic and foreign households' optimization problem. In the Appendix A, **(A.1.19)** and molding **(A.1.19)** for generic country j both of them produce¹⁹:

$$C_t = \varphi_j(C_t^j)(Q_{j,t})^{\frac{1}{\varepsilon}}$$
(1.1.58)

Log linearization:

$$c_t = c_t^j + \frac{1}{\varepsilon} q_{j,t}$$

Integrating over *j* yeilds:

$$\int_{0}^{1} c_{t} dj = \int_{0}^{1} c_{t}^{j} dj + \frac{1}{\varepsilon} \int_{0}^{1} q_{j,t} dj$$

$$c_t = \int_0^1 c_t \, dj \tag{1.1.59}$$

$$c_t^W = \int_0^1 c_t^j \, dj \tag{1.1.60}$$

Plugging (1.1.55), (1.1.59) and (1.1.60) to make:

$$c_t = c_t^W + \frac{1}{\varepsilon} q_t$$

Plugging **(1.1.57)**

¹⁹ (A.1.26) in the Appendix A makes (1.1.58).

$$c_t = c_t^W + \left(\frac{1-\alpha}{\varepsilon}\right) s_t \tag{1.1.61}$$

5 Firms

Indian economy comprises of two sectors of production²⁰ i.e. domestic formal sector and domestic informal sector. Again domestic formal sector of production is made of three types of firms: final goods producing firms, intermediate goods producing firms and importers. The final goods producing firms buy the domestic intermediate varieties produced by domestic intermediate goods producing firms and assemble them as domestically produced final goods. These firms sell a portion of their goods in the domestic formal sector goods market and export the rest. Importers²¹ on the other hand purchase foreign produced goods at world market prices and sell them in the domestic formal sector goods market which are finally consumed by the domestic consumers. The domestic informal sector production is comprises of two types of firms: final goods producing firms and intermediate goods producing firms. In this sector there are no exporting or importing firms. The final goods producing informal sector firms work in a very similar pattern as that of the final goods producing formal sector firms. But the point of deviation is that they buy only the domestic informal sector intermediate varieties produced by domestic informal sector intermediate goods producing firms and assemble them as domestically produced final goods. The whole of final goods produced in informal sector are consumed by the domestic consumers.

The CES aggregator of domestically produced goods Y_t , analogous to **(1.1.6)**, can be given as following Dixit and Stiglitz (1977):

$$Y_{t} = \left[(\gamma)^{\frac{1}{\vartheta_{b}}} (Y_{HF,t})^{\frac{\vartheta_{b}-1}{\vartheta_{b}}} + (1-\gamma)^{\frac{1}{\vartheta_{b}}} (Y_{HI,t})^{\frac{\vartheta_{b}-1}{\vartheta_{b}}} \right]^{\frac{\vartheta_{b}}{\vartheta_{b}-1}}$$
(1.2.1)

²⁰ Firms (production) strictly follow(s) the assumptions of households (consumption).

Importers are kept out of study as model is defined to target domestic inflation only.

Where $Y_{HF,t}$ and $Y_{HI,t}$ are indices of production of domestic formal and informal sector produced goods, respectively and ϑ_b is intratemporal elasticity of substitution of production between formal and informal sector produced goods. γ and $1-\gamma$ are share of domestic production of formal and informal sector produced goods, respectively. The CES function of production of domestic formal sector produced goods $Y_{HF,t}$, analogous to **(1.1.7)** can be given as:

$$Y_{HF,t} = \left[\int_{0}^{1} \left[Y_{HF,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(1.2.2)

Where $[Y_{HF,t}(i)]$ is the quantity of good i produced in the domestic formal sector in period t and θ_c is intratemporal elasticity of substitution of production between varieties of formal sector produced goods. The CES function of production of domestic informal sector produced goods $Y_{HI,t}$, analogous to (1.1.8) can be given as:

$$Y_{HI,t} = \left[\int_{0}^{1} \left[Y_{HI,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(1.2.3)

Where $[Y_{HI,t}(i)]$ is the quantity of good i produced in the domestic informal sector in period t and ϑ_c is intratemporal elasticity of substitution of production between varieties of informal sector produced goods. The corresponding consumption based price indices of (1.2.1) to (1.2.3) are given by (1.2.4) to (1.2.6), analogous to (1.1.12) to (1.1.14) as:

$$P_{H,t} = \left[\gamma (P_{HF,t})^{1-\vartheta_b} + (1-\gamma) (P_{HI,t})^{1-\vartheta_b} \right]^{\frac{1}{1-\vartheta_b}}$$
 (1.2.4)

$$P_{HF,t} = \left[\int_{0}^{1} \left[P_{HF,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
 (1.2.5)

$$P_{HI,t} = \left[\int_{0}^{1} \left[P_{HI,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
 (1.2.6)

Optimal allocation of goods derives the following demand functions in each category of production for given level of expenditure as²²:

$$Y_{HF,t} = \gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} Y_t \tag{1.2.7}$$

$$Y_{HI,t} = (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b} Y_t$$
 (1.2.8)

$$[Y_{HF,t}(i)] = \left(\frac{[P_{HF,t}(i)]}{P_{HF,t}}\right)^{-\vartheta_c} Y_{HF,t}$$
(1.2.9)

$$[Y_{HI,t}(i)] = \left(\frac{[P_{HI,t}(i)]}{P_{HI,t}}\right)^{-\vartheta_c} Y_{HI,t}$$
(1.2.10)

Intermediate goods producing firms of both formal and informal sector work in very similar fashion, they both use only labour as the key input to produce the intermediate goods. The capital stock is treated as fixed and investment is set to zero in short run following McCallum and Nelson (1999), who argue that, for most monetary policy and business-cycle analyses, fluctuations in the stock of capital do not play a major role. Walsh (2010) is, also, of the same view that capital stock be ignored in the short run as variation in capital stock does not have any significant effect on output.

²² (1.2.7) to (1.2.10) are given, analogously, to (1.1.19) to (1.1.22).

The intermediate goods producing firms use the following Cobb-Douglas technology to produce (i) type of good.

$$[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)]^{1-\phi}(K_{HI,t})^{\phi}$$
(1.2.11)

Where $[Y_{HI,t}(i)]$ is the quantity of type (i) good produced in the informal sector. $A_{HI,t}$ is the state of technology used, evenly, throughout in the informal sector and $K_{HI,t}$ stands for the capital stock²³ used. In the short run capital stock remains unchanged, i.e. $\phi = 0$, therefore:

$$[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)]^{1-0}(K_{HI,t})^{0}$$

$$[Y_{HI,t}(i)] = A_{HI,t}[L_{HI,t}(i)] \forall i \in [0,1]$$
 (1.2.12)

$$[Y_{HI,t}] = A_{HI,t}[L_{HI,t}]$$
 (1.2.13)

(1.2.13) tells a story that intermediate goods producing firms use a linear technology to produce the intermediate goods in the informal sector. Formal sector counterparts of informal sector (1.2.14) and (1.2.15) can be given, analogous to (1.2.12) and (1.2.13), as:

$$[Y_{HF,t}(i)] = A_{HF,t}[L_{HF,t}(i)] \forall i \in [0,1]$$
 (1.2.14)

$$[Y_{HF,t}] = A_{HF,t}[L_{HF,t}]$$
 (1.2.15)

Where $[Y_{HF,t}(i)]$ is the quantity of type (i) good produced in the formal sector. $A_{HF,t}$ is the state of technology used, evenly, throughout in the informal sector.

Domestic formal sector firms set prices staggering à la Calvo (1983). Each domestic formal sector firm may reset its price only with probability $(1 - \theta_{HF})$ in any given period, independent of the time elapsed since the last adjustment.

Capital stock are assumed to be constant in the short run, it follows zero investment.

Thus, each period a measure $(1 - \theta_{HF})$ of domestic formal sector firms reset their prices, while a fraction θ_{HF} keep their prices unchanged. As a result, the average duration of a price is given by $\frac{1}{(1-\theta_{HF})}$. In this context, θ_{HF} becomes a natural index of price stickiness in the domestic formal sector, Gali (2008). The domestic formal sector price dynamics (inflation) can be given as²⁴:

$$\pi_{HF,t} = (1 - \theta_{HF}) (p_{HF,t}^* - p_{HF,t-1})$$
 (1.2.16)

Where $p_{HF,t}^*$ is the optimal price set by domestic formal sector firms who are able to re-optimize in the period t. (1.2.16) makes it clear that domestic formal sector inflation results from the fact that firms in this sector re-optimizing in any given period choose a price that differs from the sector's average price in the previous period.

5.1 Optimal Price Setting

A domestic formal sector representative firm who re-optimizing in period t will choose the price $p_{HF,t}^*$ that maximizes the current market value of the profits generated while that price remains effective and the formal sector representative firm's profit maximization problem can be given as:

$$\underbrace{Max}_{\{P_{HF,t}^*\}} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left\{ \left(P_{HF,t}^* \right) \left(Y_{HF,t+k|t}(i) \right) - \left(T C_{HF,t+k|t}^N(i) \right) \left(Y_{HF,t+k|t}(i) \right) \right\} \right]$$
(1.2.17)

Subject to

$$\left(Y_{HF,t+k|t}(i)\right) = \left(\frac{P_{HF,t}^*}{P_{HF,t+k}}\right)^{-\vartheta_c} C_{HF,t+k}$$

 $^{^{24}}$ (A.2.3) in the Appendix A makes (1.2.16).

 $P_{HF.t}^*$

$$= \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) \frac{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(P_{HF,t+k}\right)^{\vartheta_c} \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(M C_{HF,t+k}^R\right)\right]}{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(P_{HF,t+k}\right)^{\vartheta_c - 1} \left(C_{HF,t+k}\right)^{1-\varepsilon}\right]}$$
(1.2.18)

Where, $E_t\left(TC_{HF,t+k|t}^N(i)\right)$, is expected future nominal total cost, for time t+k, to produce domestic formal sector good (i) and $\left(MC_{HF,t+k|t}^R\right)$ is corresponding expected future nominal marginal cost. $P_{HF,t}^*$ is optimal price²⁵ of the formal sector firms and it is given as a weighted average of future real marginal cost of the formal sector.

$$P_{HF,t}^* = \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) M C_{HF,t|t}^N = \mathcal{M}_{HF} M C_{HF,t|t}^N$$
(1.2.19)

Where, $\mathcal{M}_{HF} = \left(\frac{\vartheta_c}{\vartheta_c - 1}\right)$ and **(1.2.19)** gives that \mathcal{M}_{HF} is a desired or frictionless markup²⁶ for formal sector firms.

Linearization of (1.2.18) around steady state makes:

$$p_{HF,t}^* = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left[\left(mc_{HF,t+k|t}^R - mc_{HF}^R \right) + p_{HF,t+k} \right]$$
 (1.2.20)

$$\mu_{HF} \equiv -mc_{HF}^R \tag{1.2.21}$$

Plugging (1.2.21) in (1.2.20)

$$p_{HF,t}^* = \mu_{HF} + (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left(mc_{HF,t+k|t}^R + p_{HF,t+k} \right)$$
 (1.2.22)

²⁵ (A.2.9) in the Appendix A makes (1.2.18).

²⁶ (A.2.11) in the Appendix A makes (1.2.19).

(1.2.22) shows that domestic formal sector firms resetting their prices will choose a price that corresponds to the desired markup over a weighted average of their current and expected future real marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon $(\theta_{HF})^k$.

5.2 New Keynesian Phillips Curve for Domestic Formal Sector

Formal sector nominal wage is defined as:

$$MC_{HF,t}(i)MPL_{HF,t}(i) = W_{HF,t}$$
 (1.2.23)

Where $MPL_{HF,t}$ is marginal production of labour and $MC_{HF,t}$ nominal marginal cost.

$$\frac{MC_{HF,t}(i)}{P_{HF,t}}MPL_{HF,t}(i) = \frac{W_{HF,t}}{P_{HF,t}}$$

$$MC_{HF,t}^{R}(i) = \frac{MC_{HF,t}(i)}{P_{HF,t}}$$
 (1.2.24)

Plugging (1.2.24) in (1.2.23) makes:

$$MC_{HF,t}^{R}(i)MPL_{HF,t}(i) = \frac{W_{HF,t}}{P_{HF,t}}$$

Log linearization makes:

$$mc_{HF,t}^{R}(i) = w_{HF,t} - p_{HF,t} - mpl_{HF,t}(i)$$
 (1.2.25)

Rewriting (1.2.14)

$$[Y_{HF,t}(i)] = A_{HF,t}[L_{HF,t}(i)]$$

$$\frac{d}{di}\big[Y_{HF,t}(i)\big] = \frac{d}{di}\big(A_{HF,t}\big[L_{HF,t}(i)\big]\big)$$

$$MPL_{HF,t}(i) = A_{HF,t}$$

Log linearization makes:

$$mpl_{HF,t}(i) = a_{HF,t}$$
 (1.2.26)

Substituting (1.2.26) in (1.2.25)

$$mc_{HF,t}^{R}(i) = w_{HF,t} - p_{HF,t} - a_{HF,t}$$
 (1.2.27)

(1.2.27) shows that $mc_{HF,t}^R$ is independent of production level and uniform across all firms of the domestic formal sector, thus **(1.2.27)** becomes:

$$mc_{HF,t}^{R} = w_{HF,t} - p_{HF,t} - a_{HF,t}$$
 (1.2.28)

A similar expression to **(1.2.27)** for period t + k is given as:

$$mc_{HF,t+k|t}^{R}(i) = w_{HF,t+k} - p_{HF,t+k} - a_{HF,t+k}$$
 (1.2.29)

A similar expression to (1.2.28) for period t + k is given as:

$$mc_{HF,t+k}^{R} = w_{HF,t+k} - p_{HF,t} - a_{HF,t+k}$$
 (1.2.30)

(1.2.29) and (1.2.30) make:

$$mc_{HF,t+k|t}^{R}(i) = mc_{HF,t+k}^{R}$$
 (1.2.31)

Rewriting (1.2.20)

$$p_{HF,t}^* = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k [(mc_{HF,t+k|t}^R - mc_{HF}^R) + p_{HF,t+k}]$$

$$p_{HF,t}^* \equiv p_{HF,t+k|t}$$
 (1.2.32)

Using (1.2.32)

$$\begin{aligned} p_{HF,t}^* - p_{HF,t-1} \\ &= (1 \\ &- \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \big[\big(m c_{HF,t+k|t}^R - m c_{HF}^R \big) + p_{HF,t+k} - p_{HF,t-1} \big] \end{aligned}$$

Inserting (1.2.31)

$$p_{HF,t}^* - p_{HF,t-1}$$

$$= (1)$$

$$-\theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \Big[\Big(mc_{HF,t+k|t}^R(i) - mc_{HF}^R \Big) + p_{HF,t+k}$$

$$-p_{HF,t-1} \Big]$$

$$\widehat{mc}_{HF,t+k}^R = mc_{HF,t+k|t}^R(i) - mc_{HF}^R$$
(1.2.33)

Plugging (1.2.33)

$$p_{HF,t}^* - p_{HF,t-1} = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \left[\widehat{mc}_{HF,t+k}^R + p_{HF,t+k} - p_{HF,t-1} \right]$$

$$\begin{split} p_{HF,t}^* - p_{HF,t-1} \\ &= (1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k}^R \\ &+ (1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \big[p_{HF,t+k} - p_{HF,t-1} \big] \end{split}$$

$$(1 - \theta_{HF}\beta)E_{t} \sum_{k=0}^{\infty} (\theta_{HF})^{k} \beta^{k} [p_{HF,t+k} - p_{HF,t-1}]$$

$$= E_{t} \sum_{k=0}^{\infty} (\theta_{HF})^{k} \beta^{k} \pi_{HF,t+k}$$
(1.2.34)

Plugging (1.2.34)

$$p_{HF,t}^* - p_{HF,t-1} = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k}^R + E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k}$$

Rewriting the expression by taking k=0 out from the summation operator as:

$$\begin{split} p_{HF,t}^* - p_{HF,t-1} \\ &= (1 - \theta_{HF}\beta) E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k}^R \\ &+ (1 - \theta_{HF}\beta) (\theta_{HF})^0 \beta^0 \widehat{mc}_{HF,t+0}^R + E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \\ &+ (\theta_{HF})^0 \beta^0 \pi_{HF,t+0} \end{split}$$

$$\begin{split} p_{HF,t}^* - p_{HF,t-1} \\ &= (1 - \theta_{HF}\beta)E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k}^R + E_t \sum_{k=1}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k} \\ &+ (1 - \theta_{HF}\beta)\widehat{mc}_{HF,t}^R + \pi_{HF,t} \end{split}$$

$$\begin{split} p_{HF,t}^* - p_{HF,t-1} \\ &= (\theta_{HF}) \beta \left[(1 - \theta_{HF} \beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k+1}^R \right. \\ &+ E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k+1} \right] + (1 - \theta_{HF} \beta) \widehat{mc}_{HF,t}^R + \pi_{HF,t} \end{split}$$

$$p_{HF,t+1}^* - p_{HF,t-1+1}$$

$$= \left[(1 - \theta_{HF}\beta) E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \widehat{mc}_{HF,t+k+1}^R + E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \beta^k \pi_{HF,t+k+1} \right]$$
(1.2.35)

Plugging (1.2.35)

$$p_{HF,t}^* - p_{HF,t-1} = (\theta_{HF})\beta (p_{HF,t+1}^* - p_{HF,t}) + (1 - \theta_{HF}\beta)\widehat{mc}_{HF,t}^R + \pi_{HF,t}$$

(1.2.16) writes as:

$$E_t \pi_{HF,t+1} = (1 - \theta_{HF}) \left(p_{HF,t+1}^* - p_{HF,t} \right)$$

$$\left(p_{HF,t+1}^* - p_{HF,t} \right) = \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})}$$
(1.2.36)

Inserting (1.2.36)

$$p_{HF,t}^* - p_{HF,t-1} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF}\beta) \widehat{mc}_{HF,t}^R + \pi_{HF,t}$$

Substituting (1.2.16)

$$\frac{\pi_{HF,t}}{(1-\theta_{HF})} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1-\theta_{HF})} + (1-\theta_{HF}\beta) \widehat{mc}_{HF,t}^R + \pi_{HF,t}$$

$$\frac{\pi_{HF,t}}{(1 - \theta_{HF})} - \pi_{HF,t} = (\theta_{HF})\beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF}\beta) \widehat{mc}_{HF,t}^R$$

$$\theta_{HF} \frac{\pi_{HF,t}}{(1 - \theta_{HF})} = (\theta_{HF}) \beta \frac{E_t \pi_{HF,t+1}}{(1 - \theta_{HF})} + (1 - \theta_{HF} \beta) \widehat{mc}_{HF,t}^R$$

$$\pi_{HF,t} = \beta E_t \pi_{HF,t+1} + \frac{(1 - \theta_{HF})(1 - \theta_{HF}\beta)}{\theta_{HF}} \widehat{mc}_{HF,t}^R$$

$$\lambda_{HF} = \frac{(1 - \theta_{HF})(1 - \theta_{HF}\beta)}{\theta_{HF}}$$
 (1.2.37)

Plugging (1.2.37)

$$\pi_{HF,t} = \beta E_t \pi_{HF,t+1} + \lambda_{HF} \widehat{mc}_{HF,t}^R$$
 (1.2.38)

(1.2.38) is a New Keynesian Phillips Curve for domestic formal sector. New Keynesian Phillips Curve constitutes one of the key building blocks of New Keynesian Model. Informal sector counterparts to formal sector **(1.2.38)** and **(1.2.37)** are, analogously, given by **(1.2.39)** and **(1.2.40)**, respectively.

$$\pi_{HI,t} = \beta E_t \pi_{HI,t+1} + \lambda_{HI} \widehat{mc}_{HI,t}^R$$
 (1.2.39)

$$\lambda_{HI} = \frac{(1 - \theta_{HI})(1 - \theta_{HI}\beta)}{\theta_{HI}}$$
 (1.2.40)

6 Equilibrium Dynamics

6.1 Equilibrium in Domestic Goods Market

Market clearing for the domestic goods market requires:

$$[Y_t] = [Y_{HF,t}] + [Y_{HI,t}] = [C_{HF,t}] + [C_{HI,t}] + \int_0^1 C_{H,t}^j dj$$
 (1.3.1)

Where $C_{H,t}^j$ is the consumption of domestically produced goods in the country j and $\int_0^1 C_{H,t}^j \, dj$ is consumption across the globe for domestically produced goods. Plugging **(A.3.3)**, **(A.3.6)** and **(A.3.17)** from Appendix A in **(1.3.1)** make as:

$$\begin{split} Y_t &= \left[\gamma (1 - \alpha) \left(\frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \right] \\ &+ \left[(1 - \alpha) (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \right] \\ &+ \int_0^1 \left[\alpha \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^j} \right)^{-\vartheta_d} \left(\frac{P_{F,t}^j}{P_t^j} \right)^{-\vartheta_a} C_t^j \right] dj \end{split}$$

$$\begin{split} Y_t &= (1-\alpha)C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} + (1-\gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b}\right) \\ &+ \alpha \int\limits_0^1 C_t^j \left(P_{H,t}\right)^{-\vartheta_d} \left(\epsilon_{j,t} P_{F,t}^j\right)^{\vartheta_d} \left(P_{F,t}^j\right)^{-\vartheta_a} \left(P_t^j\right)^{\vartheta_a} dj \end{split}$$

$$Y_{t} = (1 - \alpha)C_{t} \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma)\left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}}\right)$$
$$+ \alpha \int_{0}^{1} C_{t}^{j} \left(P_{H,t}\right)^{-\vartheta_{d}} \left(P_{F,t}^{j}\right)^{\vartheta_{d}-\vartheta_{a}} \left(\epsilon_{j,t}\right)^{\vartheta_{d}} \left(P_{t}^{j}\right)^{\vartheta_{a}} dj$$

$$\begin{split} Y_t &= (1-\alpha)C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} + (1-\gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b}\right) \\ &+ \alpha \int\limits_0^1 C_t^j \left(P_{H,t}\right)^{-\vartheta_d} \left(P_{F,t}^j\right)^{\vartheta_d - \vartheta_a} \left(\epsilon_{j,t}\right)^{\vartheta_d} \left(P_t^j\right)^{\vartheta_a} \left[\frac{\epsilon_{j,t} P_t^j}{P_t}\right]^{-\vartheta_a} \left[\frac{\epsilon_{j,t} P_t^j}{P_t}\right]^{\vartheta_a} dj \end{split}$$

$$\begin{split} Y_t &= (1-\alpha)C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} + (1-\gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b}\right) \\ &+ \alpha \int\limits_0^1 C_t^j \left(P_{H,t}\right)^{-\vartheta_d} \left(P_{F,t}^j\right)^{\vartheta_d - \vartheta_a} \left(\epsilon_{j,t}\right)^{\vartheta_d} \left(\epsilon_{j,t}\right)^{-\vartheta_a} (P_t)^{\vartheta_a} \left[\frac{\epsilon_{j,t} P_t^j}{P_t}\right]^{\vartheta_a} \, dj \end{split}$$

Inserting (1.1.53)

$$Y_{t} = (1 - \alpha)C_{t} \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma)\left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}}\right)$$
$$+ \alpha \int_{0}^{1} C_{t}^{j} \left(P_{H,t}\right)^{-\vartheta_{d}} \left(\epsilon_{j,t} P_{F,t}^{j}\right)^{\vartheta_{d} - \vartheta_{a}} (P_{t})^{\vartheta_{a}} \left[Q_{j,t}\right]^{\vartheta_{a}} dj$$

$$\begin{split} Y_t &= (1-\alpha)C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} + (1-\gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b}\right) \\ &+ \alpha \int\limits_0^1 C_t^j \left(P_{H,t}\right)^{-\vartheta_d} \frac{\left(P_{H,t}\right)^{-\vartheta_a}}{\left(P_{H,t}\right)^{-\vartheta_a}} \left(\epsilon_{j,t} P_{F,t}^j\right)^{\vartheta_d - \vartheta_a} \left(\frac{1}{P_t}\right)^{-\vartheta_a} \left[\mathcal{Q}_{j,t}\right]^{\vartheta_a} dj \end{split}$$

$$Y_{t} = (1 - \alpha)C_{t} \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma)\left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}}\right)$$

$$+ \alpha \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \int_{0}^{1} C_{t}^{j} \left(P_{H,t}\right)^{-\vartheta_{d}} \frac{1}{\left(P_{H,t}\right)^{-\vartheta_{a}}} \left(\epsilon_{j,t} P_{F,t}^{j}\right)^{\vartheta_{d}-\vartheta_{a}} \left[Q_{j,t}\right]^{\vartheta_{a}} dj$$

$$Y_{t} = \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left[(1 - \alpha)C_{t} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}} \right) + \alpha \int_{0}^{1} C_{t}^{j} \left(\frac{\epsilon_{j,t}P_{F,t}^{j}}{P_{H,t}}\right)^{\vartheta_{d}-\vartheta_{a}} \left[Q_{j,t}\right]^{\vartheta_{a}} dj \right]$$

Plugging **(1.1.58)** by assuming $(Q_{j,t})^{-\frac{1}{\varepsilon}}C_t \cong C_t^j$

$$Y_{t} = \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left[(1 - \alpha)C_{t} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}} \right) + \alpha \int_{0}^{1} \left(\frac{\epsilon_{j,t}P_{F,t}^{j}}{P_{H,t}}\right)^{\vartheta_{d}-\vartheta_{a}} \left[Q_{j,t}\right]^{\vartheta_{a}} (Q_{j,t})^{-\frac{1}{\varepsilon}} C_{t} dj \right]$$

$$Y_{t} = \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left[(1 - \alpha)C_{t} \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}} \right) + \alpha C_{t} \int_{0}^{1} \left(\frac{\epsilon_{j,t} P_{F,t}^{j}}{P_{H,t}}\right)^{\vartheta_{d} - \vartheta_{a}} \left[Q_{j,t}\right]^{\vartheta_{a} - \frac{1}{\varepsilon}} dj \right]$$

$$\begin{split} Y_t &= C_t \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} \left[(1-\alpha) \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} + (1-\gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b} \right) \right. \\ &+ \alpha \int\limits_0^1 \left(\frac{\epsilon_{j,t} P_{F,t}^j}{P_{j,t}} \frac{P_{j,t}}{P_{H,t}} \right)^{\vartheta_d - \vartheta_a} \left[\mathcal{Q}_{j,t} \right]^{\vartheta_a - \frac{1}{\varepsilon}} dj \right] \end{split}$$

Plugging **(1.1.39)** and $S_t^j = \frac{\epsilon_{j,t} P_{F,t}^j}{P_{j,t}}$

$$Y_{t} = C_{t} \left(\frac{P_{H,t}}{P_{t}}\right)^{-\vartheta_{a}} \left[(1 - \alpha) \left(\gamma \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_{b}} + (1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_{b}} \right) + \alpha \int_{0}^{1} \left(S_{t}^{j} S_{j,t} \right)^{\vartheta_{d} - \vartheta_{a}} \left[Q_{j,t} \right]^{\vartheta_{a} - \frac{1}{\varepsilon}} dj \right]$$

$$(1.3.2)$$

Log Linearization of **(1.3.2)** at steady state $P_{HF,t} = P_{HI,t} = P_{H,t} = P_{t} = P_{t}$

$$y_t = c_t + \alpha \vartheta_d s_t + \alpha \left(\vartheta_a - \frac{1}{\varepsilon}\right) q_t$$
 (1.3.3)

Substituting (1.1.57) in (1.3.3)

$$y_{t} = c_{t} + \alpha \vartheta_{d} s_{t} + \alpha \left(\vartheta_{a} - \frac{1}{\varepsilon}\right) (1 - \alpha) s_{t}$$

$$y_{t} = c_{t} + \alpha s_{t} \left[\vartheta_{d} + \left(\vartheta_{a} - \frac{1}{\varepsilon}\right) (1 - \alpha)\right]$$

$$y_{t} = c_{t} + \alpha s_{t} \left[\vartheta_{d} + \left(\vartheta_{a} - \frac{1}{\varepsilon}\right) (1 - \alpha)\right] \frac{\varepsilon}{\varepsilon}$$

$$y_{t} = c_{t} + \alpha s_{t} \frac{\left[\varepsilon \vartheta_{d} + \left(\varepsilon \vartheta_{a} - 1\right) (1 - \alpha)\right]}{\varepsilon}$$

$$\omega = \left[\varepsilon \vartheta_{d} + \left(\varepsilon \vartheta_{a} - 1\right) (1 - \alpha)\right]$$

$$(1.3.4)$$

Plugging (1.3.4) in the expression makes:

$$y_t = c_t + \alpha s_t \frac{\omega}{\varepsilon} \tag{1.3.5}$$

Following **(1.3.5)** the total output of a generic country *j* can be written as:

$$y_t^j = c_t^j + \alpha \frac{\omega}{\varepsilon} s_t^j$$
 (1.3.6)

The total output of the world can be written as:

$$y_t^w = \int_0^1 y_t^j dj = \int_0^1 \left(c_t^j + \alpha \frac{\omega}{\varepsilon} s_t^j \right) dj$$

$$y_t^w = \int_0^1 (c_t^j) \, dj + \alpha \frac{\omega}{\varepsilon} \int_0^1 (s_t^j) \, dj$$

Plugging **(1.1.60)**

$$y_t^w = c_t^w + \alpha \frac{\omega}{\varepsilon} \int_0^1 (s_t^j) \, dj$$

$$\int_{0}^{1} \left(s_{t}^{j} \right) dj = 0 \tag{1.3.7}$$

Inserting **(1.3.7)**

$$y_t^w = c_t^w \tag{1.3.8}$$

The total output of the world is equal to the total consumption of the world.

Inserting (1.1.61) in (1.3.5)

$$y_t = c_t^w + \frac{(1-\alpha)}{\varepsilon} s_t + \alpha s_t \frac{\omega}{\varepsilon}$$

$$y_t = c_t^w + \frac{s_t}{\varepsilon} (1 - \alpha + \alpha \omega)$$

Inserting (1.3.8)

$$y_t = y_t^w + \frac{s_t}{\varepsilon} (1 - \alpha + \alpha \omega)$$

$$y_t = y_t^w + \frac{s_t}{\varepsilon}$$

$$\frac{s_t}{(1 - \alpha + \alpha \omega)}$$

Assume

$$\varepsilon_{\alpha} = \frac{\varepsilon}{(1 - \alpha + \alpha\omega)} \tag{1.3.9}$$

Plugging (1.3.9)

$$y_t = y_t^w + \frac{s_t}{\varepsilon_\alpha} \tag{1.3.10}$$

Substituting (1.3.5) in the consumption Euler (1.1.37) make:

$$y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t c_{t+1} + \frac{1}{\varepsilon} (\mathfrak{r} - i_t + E_t \pi_{t+1})$$

(1.3.5) makes:

$$y_{t+1} - \alpha s_{t+1} \frac{\omega}{\varepsilon} = c_{t+1}$$
 (1.3.11)

Inserting (1.3.11)

$$y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t \left[y_{t+1} - \alpha s_{t+1} \frac{\omega}{\varepsilon} \right] + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1})$$

$$y_t - \alpha s_t \frac{\omega}{\varepsilon} = E_t y_{t+1} - \frac{\omega}{\varepsilon} \alpha E_t s_{t+1} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1})$$

$$y_t = E_t y_{t+1} - \frac{\omega}{\varepsilon} \alpha E_t s_{t+1} + \alpha s_t \frac{\omega}{\varepsilon} + \frac{1}{\varepsilon} (r - i_t + E_t \pi_{t+1})$$

$$y_t = E_t y_{t+1} + \frac{1}{\varepsilon} (\mathbf{r} - i_t + E_t \pi_{t+1}) - \frac{\omega}{\varepsilon} \alpha E_t \Delta s_{t+1}$$
 (1.3.12)

(1.3.12) is an IS Equation and can be represented in some other forms as follows:

(1.1.45) makes:

$$\pi_{t+1} = \pi_{H,t+1} + \alpha \Delta s_{t+1} \tag{1.3.13}$$

Substituting (1.3.13) in (1.3.12)

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon}\left(\mathbf{r} - i_{t} + E_{t}\left[\pi_{H,t+1} + \alpha\Delta s_{t+1}\right]\right) - \frac{\omega}{\varepsilon}\alpha E_{t}\Delta s_{t+1}$$

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon}\left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1} + E_{t}\alpha\Delta s_{t+1}\right) - \frac{\omega}{\varepsilon}\alpha E_{t}\Delta s_{t+1}$$

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon}\left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1}\right) + \frac{1}{\varepsilon}\left(E_{t}\alpha\Delta s_{t+1}\right) - \frac{\omega}{\varepsilon}\alpha E_{t}\Delta s_{t+1}$$

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon}\left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1}\right) - \left(\frac{\omega - 1}{\varepsilon}\right)\alpha E_{t}\Delta s_{t+1}$$

Assume

$$(\omega - 1) = \Theta \tag{1.3.14}$$

Plugging (1.3.14) to make:

$$y_t = E_t y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1} \right) - \frac{\Theta}{\varepsilon} \alpha E_t \Delta s_{t+1}$$
 (1.3.15)

(1.3.15) is an another version of IS equation.

Rewriting (1.3.10)

$$y_t = y_t^w + \frac{s_t}{\varepsilon_\alpha}$$

$$\varepsilon_\alpha(y_t - y_t^w) = s_t$$
(1.3.16)

$$\varepsilon_{\alpha}(y_{t+1} - y_{t+1}^{w}) = s_{t+1}$$
(1.3.17)

$$\Delta s_{t+1} = s_{t+1} - s_t \tag{1.3.18}$$

Plugging (1.3.17) and (1.3.16) in (1.3.18)

$$\Delta s_{t+1} = \varepsilon_{\alpha} (y_{t+1} - y_{t+1}^{w}) - \varepsilon_{\alpha} (y_{t} - y_{t}^{w})$$

$$\Delta s_{t+1} = \varepsilon_{\alpha} \{ (y_{t+1} - y_{t+1}^{w}) - (y_{t} - y_{t}^{w}) \}$$
(1.3.19)

Plugging (1.3.19) in (1.3.15)

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1} \right) - \frac{\Theta}{\varepsilon} \alpha E_{t} \left[\varepsilon_{\alpha} \left\{ (y_{t+1} - y_{t+1}^{w}) - (y_{t} - y_{t}^{w}) \right\} \right]$$

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1} \right) - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon} E_{t} \left[y_{t+1} - y_{t+1}^{w} - y_{t} + y_{t}^{w} \right]$$

$$y_{t} = E_{t}y_{t+1} - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon} E_{t}y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1} \right) + \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon} E_{t} \left[y_{t+1}^{w} - y_{t}^{w} + y_{t} \right]$$

$$\Delta y_{t+1}^{w} = y_{t+1}^{w} - y_{t}^{w}$$

$$(1.3.20)$$

Plugging (1.3.20)

$$\begin{split} y_t &= \left(1 - \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon}\right) E_t y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon} E_t [\Delta y_{t+1}^w + y_t] \\ \\ y_t &= \left(1 - \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon}\right) E_t y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon} E_t \Delta y_{t+1}^w + \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon} y_t \\ \\ \left(1 - \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon}\right) y_t &= \left(1 - \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon}\right) E_t y_{t+1} + \frac{1}{\varepsilon} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\Theta\alpha\varepsilon_\alpha}{\varepsilon} E_t \Delta y_{t+1}^w \end{split}$$

$$y_{t} = \frac{\left(1 - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon}\right)}{\left(1 - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon}\right)} E_{t} y_{t+1} + \frac{\frac{1}{\varepsilon}}{\left(1 - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon}\right)} \left(r - i_{t} + E_{t} \pi_{H,t+1}\right) + \frac{\frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon}}{\left(1 - \frac{\Theta\alpha\varepsilon_{\alpha}}{\varepsilon}\right)} E_{t} \Delta y_{t+1}^{w}$$

$$y_t = E_t y_{t+1} + \frac{\frac{1}{\varepsilon}}{\left(1 - \frac{\Theta \alpha \varepsilon_{\alpha}}{\varepsilon}\right)} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\frac{\Theta \alpha \varepsilon_{\alpha}}{\varepsilon}}{\left(1 - \frac{\Theta \alpha \varepsilon_{\alpha}}{\varepsilon}\right)} E_t \Delta y_{t+1}^w$$

$$y_t = E_t y_{t+1} + \frac{\frac{1}{\varepsilon}}{\left(\frac{\varepsilon - \Theta \alpha \varepsilon_{\alpha}}{\varepsilon}\right)} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\frac{\Theta \alpha \varepsilon_{\alpha}}{\varepsilon}}{\left(\frac{\varepsilon - \Theta \alpha \varepsilon_{\alpha}}{\varepsilon}\right)} E_t \Delta y_{t+1}^w$$

$$y_t = E_t y_{t+1} + \frac{1}{(\varepsilon - \Theta \alpha \varepsilon_{\alpha})} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1} \right) + \frac{\Theta \alpha \varepsilon_{\alpha}}{(\varepsilon - \Theta \alpha \varepsilon_{\alpha})} E_t \Delta y_{t+1}^w$$

Plugging (1.3.9) and (1.3.14)

$$\begin{split} y_t &= E_t y_{t+1} + \frac{1}{\left(\varepsilon - (\omega - 1)\alpha \frac{\varepsilon}{(1 - \alpha + \alpha \omega)}\right)} \Big(r - i_t + E_t \pi_{H, t+1}\Big) \\ &+ \frac{(\omega - 1)\alpha \frac{\varepsilon}{(1 - \alpha + \alpha \omega)}}{\left(\varepsilon - (\omega - 1)\alpha \frac{\varepsilon}{(1 - \alpha + \alpha \omega)}\right)} E_t \Delta y_{t+1}^w \end{split}$$

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\left(\varepsilon - \frac{(\omega - 1)\alpha\varepsilon}{(1 - \alpha + \alpha\omega)}\right)} \left(r - i_{t} + E_{t}\pi_{H,t+1}\right)$$

$$+ \frac{\alpha\varepsilon(\omega - 1)}{\left(1 - \alpha + \alpha\omega\right)} E_{t}\Delta y_{t+1}^{w}$$

$$\left(\varepsilon - \frac{(\omega - 1)\alpha\varepsilon}{(1 - \alpha + \alpha\omega)}\right)$$

$$\begin{aligned} y_t &= E_t y_{t+1} + \frac{1}{\left(\frac{\varepsilon(1-\alpha+\alpha\omega)-(\omega-1)\alpha\varepsilon}{(1-\alpha+\alpha\omega)}\right)} \Big(\mathbf{r} - i_t + E_t \pi_{H,t+1}\Big) \\ &+ \frac{\alpha\varepsilon(\omega-1)}{\left(1-\alpha+\alpha\omega\right)} \\ &+ \frac{\left(\frac{\varepsilon(1-\alpha+\alpha\omega)-(\omega-1)\alpha\varepsilon}{(1-\alpha+\alpha\omega)}\right)}{\left(\frac{\varepsilon(1-\alpha+\alpha\omega)-(\omega-1)\alpha\varepsilon}{(1-\alpha+\alpha\omega)}\right)} E_t \Delta y_{t+1}^w \end{aligned}$$

$$y_t = E_t y_{t+1} + \frac{1}{\left(\frac{\varepsilon}{(1-\alpha+\alpha\omega)}\right)} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1}\right) + \frac{\frac{\alpha\varepsilon(\omega-1)}{(1-\alpha+\alpha\omega)}}{\left(\frac{\varepsilon}{(1-\alpha+\alpha\omega)}\right)} E_t \Delta y_{t+1}^w$$

$$y_{t} = E_{t}y_{t+1} + \frac{(1 - \alpha + \alpha\omega)}{\varepsilon} \left(\mathbf{r} - i_{t} + E_{t}\pi_{H,t+1} \right) + \frac{\alpha\varepsilon(\omega - 1)}{(1 - \alpha + \alpha\omega)} \frac{(1 - \alpha + \alpha\omega)}{\varepsilon} E_{t}\Delta y_{t+1}^{w}$$

$$y_t = E_t y_{t+1} + \frac{(1-\alpha+\alpha\omega)}{\varepsilon} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1} \right) + \alpha(\omega-1) E_t \Delta y_{t+1}^w$$

Plugging (1.3.9) and (1.3.14)

$$y_{t} = E_{t}y_{t+1} + \frac{1}{\varepsilon_{\alpha}} \left(r - i_{t} + E_{t}\pi_{H,t+1} \right) + \alpha\Theta E_{t}\Delta y_{t+1}^{w}$$
 (1.3.21)

(1.3.21) represents another format of an IS equation.

6.2 The trade balance

The net trade balance is given in terms of domestic output and expressed as fraction of steady state output as:

$$NX_{t} = \frac{Y_{t} - \frac{P_{t}}{P_{H,t}}C_{t}}{Y}$$
 (1.3.22)

Where, NX_t , is net trade balance in time t and Y is defined as domestic output at steady state. Log linearization of **(1.3.22)** makes:

$$nx_t = y_t - c_t - p_t + p_{H,t} ag{1.3.23}$$

Substituting **(1.1.43)**

$$nx_{t} = y_{t} - c_{t} - p_{H,t} - \alpha s_{t} + p_{H,t}$$
$$nx_{t} = y_{t} - c_{t} - \alpha s_{t}$$

Inserting **(1.3.5)**

$$nx_{t} = \alpha s_{t} \frac{\omega}{\varepsilon} - \alpha s_{t}$$

$$nx_{t} = \alpha s_{t} \left(\frac{\omega}{\varepsilon} - 1\right)$$
 (1.3.24)

(1.2.14) writes as:

$$\left[L_{HF,t}(i) \right] = \frac{\left[Y_{HF,t}(i) \right]}{A_{HF,t}}$$

$$\int_{0}^{1} \left[L_{HF,t}(i) \right] di = \int_{0}^{1} \frac{\left[Y_{HF,t}(i) \right]}{A_{HF,t}} di$$

$$L_{HF,t} = \int_{0}^{1} \left[L_{HF,t}(i) \right] di$$
(1.3.25)

Plugging (1.3.25) and (1.2.9) in the expression to make:

$$L_{HF,t} = \frac{1}{A_{HF,t}} \int_{0}^{1} \left(\frac{\left[P_{HF,t}(i) \right]}{P_{HF,t}} \right)^{-\vartheta_{c}} Y_{HF,t} di$$
 (1.3.26)

(1.3.27) analogous to (1.3.26) writes as:

$$L_{HI,t} = \frac{1}{A_{HI,t}} \int_{0}^{1} \left(\frac{\left[P_{HI,t}(i) \right]}{P_{HI,t}} \right)^{-\vartheta_c} Y_{HI,t} di$$
 (1.3.27)

$$L_t = L_{HF,t} + L_{HI,t} {(1.3.28)}$$

Using (1.3.26), (1.3.27) and (1.3.28)

$$L_{t} = \frac{1}{A_{HF,t}} \int_{0}^{1} \left(\frac{\left[P_{HF,t}(i) \right]}{P_{HF,t}} \right)^{-\vartheta_{c}} Y_{HF,t} di + \frac{1}{A_{HI,t}} \int_{0}^{1} \left(\frac{\left[P_{HI,t}(i) \right]}{P_{HI,t}} \right)^{-\vartheta_{c}} Y_{HI,t} di$$
 (1.3.29)

Log linearization of **(1.3.29)** at steady state $[P_{HF,t}(i)] = [P_{HI,t}(i)] = P_{HF,t} = P_{HI,t} = P_{H,t} = P_t = P$

$$l_{t} = y_{HF,t} - a_{HF,t} + y_{HI,t} - a_{HI,t}$$

$$y_{t} = y_{HF,t} + y_{HI,t}$$
(1.3.30)

Assume

$$a_t = a_{HF,t} + a_{HI,t}$$
 (1.3.31)

Plugging (1.3.30) and (1.3.31) in the expression to make:

$$y_t = l_t + a_t \tag{1.3.32}$$

Domestic inflation is given by

$$\pi_{H,t} = \gamma \pi_{HF,t} + (1 - \gamma) \pi_{HI,t}$$
 (1.3.33)

Plugging (1.2.38) and (1.2.39) in (1.3.33)

$$\pi_{H,t} = \gamma \left[\beta E_t \pi_{HF,t+1} + \lambda_{HF} \widehat{mc}_{HF,t}^R\right] + (1 - \gamma) \left[\beta E_t \pi_{HI,t+1} + \lambda_{HI} \widehat{mc}_{HI,t}^R\right]$$

$$\pi_{H,t} = \gamma \beta E_t \pi_{HF,t+1} + \gamma \lambda_{HF} \widehat{mc}_{HF,t}^R + (1 - \gamma) \beta E_t \pi_{HI,t+1} + (1 - \gamma) \lambda_{HI} \widehat{mc}_{HI,t}^R$$

$$\pi_{H,t} = \beta \left[\gamma E_t \pi_{HF,t+1} + (1 - \gamma) E_t \pi_{HI,t+1}\right] + \gamma \lambda_{HF} \widehat{mc}_{HF,t}^R + (1 - \gamma) \lambda_{HI} \widehat{mc}_{HI,t}^R$$

The domestic informal sector has perfect Classical markets i.e. it has complete flexibility in prices (no stickiness in prices) and perfections in markets; it follows $\theta_{HI} = 0$ and in turn it (θ_{HI}) makes:

$$\widehat{mc}_{HLt}^R = 0 \tag{1.3.34}$$

Rewriting the domestic inflation as:

$$E_t \pi_{H,t+1} = \gamma E_t \pi_{HF,t+1} + (1 - \gamma) E_t \pi_{HI,t+1}$$
 (1.3.35)

Plugging **(1.3.34)**, **(1.3.35)** and at steady state $\widehat{mc}_{HF,t}^R = \widehat{mc}_t^R$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \gamma \lambda_{HF} \widehat{mc}_t^R$$
 (1.3.36)

(1.3.37), analogous to (1.2.28) can be given as:

$$mc_t^R = w_t - p_{H\,t} - a_t \tag{1.3.37}$$

Rewriting **(1.3.37)**

$$mc_t^R = w_t - p_t + p_t - p_{H,t} - a_t$$

Plugging (1.1.38) and (1.1.43)

$$mc_t^R = \varepsilon c_t + \nu l_t + \alpha s_t - a_t$$

Inserting (1.1.61) and (1.3.32)

$$mc_t^R = \varepsilon \left(c_t^w + \frac{1 - \alpha}{\varepsilon} s_t \right) + \nu (y_t - a_t) + \alpha s_t - a_t$$

$$mc_t^R = \varepsilon c_t^w + s_t - \alpha s_t + \nu y_t - \nu a_t + \alpha s_t - a_t$$

$$mc_t^R = \varepsilon c_t^w + s_t + \nu y_t - \nu a_t - a_t$$

$$mc_t^R = \varepsilon c_t^w + s_t + \nu y_t - a_t (\nu + 1)$$

Substituting (1.3.8)

$$mc_t^R = \varepsilon y_t^w + s_t + \nu y_t - a_t(\nu+1)$$

Plugging **(1.3.16)**

$$mc_t^R = \varepsilon y_t^W + s_t + v y_t - a_t(v+1)$$

$$mc_t^R = \varepsilon y_t^W + \varepsilon_\alpha (y_t - y_t^W) + v y_t - a_t(v+1)$$

$$mc_t^R = \varepsilon y_t^W + \varepsilon_\alpha y_t - \varepsilon_\alpha y_t^W + v y_t - a_t(v+1)$$

$$mc_t^R = \varepsilon y_t^W - \varepsilon_\alpha y_t^W + v y_t + \varepsilon_\alpha y_t - a_t(v+1)$$

$$mc_t^R = y_t^W (\varepsilon - \varepsilon_\alpha) + y_t(v + \varepsilon_\alpha) - a_t(v+1)$$

$$(1.3.38)$$

When prices are completely flexible (1.3.39), analogous to (1.2.21) can be given as:

$$mc^R = -\mu \tag{1.3.39}$$

Natural level of output is defined as y_t^n and using **(1.3.39)** a flexible prices version of **(1.3.38)** can be given as:

$$mc^{R} = -\mu = y_{t}^{w}(\varepsilon - \varepsilon_{\alpha}) + y_{t}^{n}(\nu + \varepsilon_{\alpha}) - a_{t}(\nu + 1)$$

$$y_{t}^{n}(\nu + \varepsilon_{\alpha}) = -\mu - y_{t}^{w}(\varepsilon - \varepsilon_{\alpha}) + a_{t}(\nu + 1)$$

$$y_{t}^{n} = a_{t}\frac{(\nu + 1)}{(\nu + \varepsilon_{\alpha})} - \frac{\mu}{(\nu + \varepsilon_{\alpha})} - y_{t}^{w}\frac{(\varepsilon - \varepsilon_{\alpha})}{(\nu + \varepsilon_{\alpha})}$$
(1.3.40)

Plugging (1.3.9) and (1.3.14)

$$y_t^n = a_t \frac{(\nu + 1)}{(\nu + \varepsilon_\alpha)} - \frac{\mu}{(\nu + \varepsilon_\alpha)} - y_t^w \frac{\left(\varepsilon - \frac{\varepsilon}{1 + \alpha\Theta}\right)}{(\nu + \varepsilon_\alpha)}$$

$$y_t^n = a_t \frac{(\nu + 1)}{(\nu + \varepsilon_\alpha)} - \frac{\mu}{(\nu + \varepsilon_\alpha)} - y_t^w \frac{\varepsilon (1 + \alpha \Theta) - \varepsilon}{(\nu + \varepsilon_\alpha)}$$

$$y_t^n = a_t \frac{(\nu + 1)}{(\nu + \varepsilon_\alpha)} - \frac{\mu}{(\nu + \varepsilon_\alpha)} - y_t^w \frac{\varepsilon}{1 + \alpha \Theta} \frac{\alpha \Theta}{1}$$

Again plugging (1.3.9) and (1.3.14)

$$y_t^n = a_t \frac{(\nu + 1)}{(\nu + \varepsilon_\alpha)} - \frac{\mu}{(\nu + \varepsilon_\alpha)} - y_t^w \frac{\alpha \Theta \varepsilon_\alpha}{(\nu + \varepsilon_\alpha)}$$
 (1.3.41)

Plugging (1.3.42), (1.3.43) and (1.3.44) in (1.3.41)

$$\Gamma_0 = -\frac{\mu}{(\nu + \varepsilon_\alpha)} \tag{1.3.42}$$

$$\Gamma_a = \frac{(\nu+1)}{(\nu+\varepsilon_\alpha)} \tag{1.3.43}$$

$$\Gamma_{w} = -\frac{\alpha \Theta \varepsilon_{\alpha}}{(\nu + \varepsilon_{\alpha})} \tag{1.3.44}$$

$$y_t^n = \Gamma_0 + a_t \Gamma_a + y_t^w \Gamma_w \tag{1.3.45}$$

7 New Keynesian Phillips Curve for Indian Economy

Domestic output gap can be defined as:

$$\tilde{y}_t = y_t - y_t^n \tag{1.3.46}$$

(1.3.38) and its flexible prices version make as:

$$\begin{split} mc_t^R - mc^R &= [y_t^w(\varepsilon - \varepsilon_\alpha) + y_t(\nu + \varepsilon_\alpha) - a_t(\nu + 1)] \\ &- [y_t^w(\varepsilon - \varepsilon_\alpha) + y_t^n(\nu + \varepsilon_\alpha) - a_t(\nu + 1)] \end{split}$$

$$mc_t^R - mc^R = [y_t(v + \varepsilon_\alpha)] - [y_t^n(v + \varepsilon_\alpha)]$$

$$\widehat{mc}_t^R = mc_t^R - mc^R \tag{1.3.47}$$

Substituting (1.3.47)

$$\widehat{mc}_t^R = (\nu + \varepsilon_\alpha)(y_t - y_t^n)$$
(1.3.48)

Inserting (1.3.46)

$$\widehat{mc}_t^R = (\nu + \varepsilon_\alpha)\widetilde{y}_t \tag{1.3.49}$$

Plugging (1.3.49) in (1.3.36)

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \gamma \lambda_{HF} (\nu + \varepsilon_{\alpha}) \tilde{y}_t$$

$$\rho = \gamma \lambda_{HF} (\nu + \varepsilon_{\alpha}) \tag{1.3.50}$$

Plugging (1.3.50)

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \rho \tilde{y}_t \tag{1.3.51}$$

(1.3.51) is an open economy New Keynesian Phillips Curve for Indian Economy.

8 Dynamic IS Curve for Indian Economy

The real rate of interest can be defined as:

$$i_t^R = i_t - E_t \pi_{H,t+1}$$
 (1.3.52)

The real rate of interest at its natural level can be defined as:

$$i_t^{R^n} = i_t - E_t \pi_{H,t+1}$$
 (1.3.53)

Natural level of output, analogous to (1.3.21) can be given using (1.3.53) as:

$$y_t^n = E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left(r - \left(i_t^{R^n} + E_t \pi_{H,t+1} \right) + E_t \pi_{H,t+1} \right) + \alpha \Theta E_t \Delta y_{t+1}^w$$

$$y_t^n = E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left(\mathbf{r} - i_t^{R^n} \right) + \alpha \Theta E_t \Delta y_{t+1}^w$$
(1.3.54)

(1.3.21) and (1.3.54) make as:

$$y_t - y_t^n = \left[E_t y_{t+1} + \frac{1}{\varepsilon_\alpha} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1} \right) + \alpha \Theta E_t \Delta y_{t+1}^w \right]$$
$$- \left[E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left(\mathbf{r} - i_t^{R^n} \right) + \alpha \Theta E_t \Delta y_{t+1}^w \right]$$
$$y_t - y_t^n = \left[E_t y_{t+1} + \frac{1}{\varepsilon_\alpha} \left(\mathbf{r} - i_t + E_t \pi_{H,t+1} \right) \right] - \left[E_t y_{t+1}^n + \frac{1}{\varepsilon_\alpha} \left(\mathbf{r} - i_t^{R^n} \right) \right]$$

$$E_t \tilde{y}_{t+1} = E_t y_{t+1} - E_t y_{t+1}^n$$
 (1.3.55)

Plugging (1.3.46) and (1.3.55), analogous to (1.3.46) make

$$\tilde{y}_t = \left[E_t \tilde{y}_{t+1} + \frac{1}{\varepsilon_\alpha} \left(i_t^{R^n} - i_t + E_t \pi_{H,t+1} \right) \right]$$
 (1.3.56)

(1.3.56) is Dynamic IS curve for the Indian Economy. Open economy New Keynesian Phillips Curve **(1.3.51)**, Dynamic IS Curve **(1.3.56)** and monetary policy rule (Taylor rule) **(2.2.15)**, which is derived in the next study entitled "Inflation Targeting Model for Indian Economy", are the key building blocks of the New Keynesian Model.

9 Conclusion

I have developed a New Keynesian Model for the Indian economy with heterogeneous sectors, namely, formal and informal. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector of Indian economy characterizes the complete flexibility in prices and wages and perfections in markets. The formal sector comprises of Keynesian markets while informal sector is made of pure Classical markets. Thus, Indian economy comprises of a very typical mixture of Keynesian and Classical markets.

When monetary policy is conducted (variation in money supply) by the Reserve Bank of India in such an environment then only nominal effects are seen in the informal sector markets i.e. variations in price and wage levels; but real effects are observed in the formal sector markets in short run i.e. monetary policy affects output and level of employment in short run.

This study is firstly and fore-mostly targeted to study the nature of Indian domestic inflation and thereby to study the real variables of the economy. The aggregate supply equation, the New Keynesian Phillips Curve, establishes a relationship between domestic inflation and output (gap). The New Keynesian Phillips Curve reveals that the degree of stickiness in prices in formal sector markets has a deep impact on the domestic inflation as informal sector markets are frictionless and have complete price flexibility (zero stickiness). Thus, degree of stickiness in prices in formal sector markets plays a major role to determine the domestic inflation and enables the monetary policy to stabilize formal sector output and level of employment.

Summarily, monetary policy affects the real variables of the economy in formal sector in short run while nominal variables (price and wage level) in informal sector. Thus, monetary policy in India has a very poor control on real variables of the economy in short run due to presence of huge informal sector. This conclusion is based on pure theory and may have deviation from reality. This study provides opportunities for further empirical work.

10 Limitation of the Study

I have dropped the government, money, habit formation and corruption in public spending (consumption spending and capital spending) related issues in this study to keep it simple and easily traceable without diluting the main concern of the study. This study is based on pure theory and no empirical support is supplemented.

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12 Appendix A

Derivation of expressions / equations for 1.3 Households

Consumption expenditures $C_{HF,t}P_{HF,t}$, $C_{HI,t}P_{HI,t}$, $C_{F,t}P_{F,t}$, $C_{H,t}P_{H,t}$ and $C_{t}P_{t}$ of the domestic households can be given as:

$$\int_{0}^{1} [C_{HF,t}(i)][P_{HF,t}(i)]di = C_{HF,t}P_{HF,t}$$
(A.1.1)

$$\int_{0}^{1} [C_{HI,t}(i)][P_{HI,t}(i)]di = C_{HI,t}P_{HI,t}$$
(A.1.2)

$$\int_{0}^{1} \int_{0}^{1} \left[C_{j,t}(i) \right] \left[P_{j,t}(i) \right] didj = C_{F,t} P_{F,t}$$
(A.1.3)

$$C_{H,t}P_{H,t} = C_{HF,t}P_{HF,t} + C_{HI,t}P_{HI,t}$$
 (A.1.4)

$$C_t P_t = C_{H,t} P_{H,t} + C_{F,t} P_{F,t}$$
 (A.1.5)

$$C_{t}P_{t} = \int_{0}^{1} \left[C_{HF,t}(i)\right] \left[P_{HF,t}(i)\right] di + \int_{0}^{1} \left[C_{HI,t}(i)\right] \left[P_{HI,t}(i)\right] di + \int_{0}^{1} \int_{0}^{1} \left[C_{j,t}(i)\right] \left[P_{j,t}(i)\right] di dj$$
(A.1.6)

Where **(A.1.1)** to **(A.1.5)** collapse to **(A.1.6)**.

Nominal wage incomes $L_{HF,t}W_{HF,t}$, $L_{HI,t}W_{HI,t}$ and L_tW_t of the domestic household can be given as:

$$\int_{0}^{1} [L_{HF,t}(i)][W_{HF,t}(i)]di = L_{HF,t}W_{HF,t}$$
(A.1.7)

$$\int_{0}^{1} [L_{HI,t}(i)][W_{HI,t}(i)]di = L_{HI,t}W_{HI,t}$$
(A.1.8)

$$L_t W_t = L_{HF,t} W_{HF,t} + L_{HI,t} W_{HI,t}$$
 (A.1.9)

Where (A.1.7) to (A.1.9) collapse to (A.1.10).

$$L_t W_t = \int_0^1 [L_{HF,t}(i)] [W_{HF,t}(i)] di + \int_0^1 [L_{HI,t}(i)] [W_{HI,t}(i)] di$$
 (A.1.10)

Plugging (1.1.7) in (A.1.1) makes

$$P_{HF,t}\left[\int_{0}^{1} \left[C_{HF,t}(i)\right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di\right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}} = \int_{0}^{1} \left[C_{HF,t}(i)\right] \left[P_{HF,t}(i)\right] di$$

$$\underbrace{\min_{\left[C_{HF,t}(i)\right]} P_{HF,t} \left[\int_{0}^{1} \left[C_{HF,t}(i)\right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}} - \int_{0}^{1} \left[C_{HF,t}(i)\right] \left[P_{HF,t}(i)\right] di$$

First Order Condition with respect to $[C_t^{HF}(i)]$ is given by

$$P_{HF,t} \frac{\vartheta_c}{\vartheta_c - 1} \frac{\vartheta_c - 1}{\vartheta_c} \left[\int_0^1 \left[C_{HF,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1} - 1} \left[C_{HF,t}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c} - 1} - \left[P_{HF,t}(i) \right] = 0$$

$$P_{HF,t} \left[\int_{0}^{1} \left[C_{HF,t}(i) \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{1}{\vartheta_{c}-1}} \left[C_{HF,t}(i) \right]^{\frac{-1}{\vartheta_{c}}} = \left[P_{HF,t}(i) \right]$$

$$\left[C_{HF,t}(i)\right]^{\frac{-1}{\vartheta_c}} = \frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}} \left[\int_{0}^{1} \left[C_{HF,t}(i)\right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di\right]^{\frac{-1}{\vartheta_c - 1}}$$

$$\left[C_{HF,t}(i)\right]^{\frac{\partial_{c}}{\partial_{c}}} = \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\partial_{c}} \left[\int_{0}^{1} \left[C_{HF,t}(i)\right]^{\frac{\partial_{c}-1}{\partial_{c}}} di\right]^{\frac{\partial_{c}}{\partial_{c}-1}}$$

$$\left[C_{HF,t}(i)\right] = \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\vartheta_c} \left[\int_{0}^{1} \left[C_{HF,t}(i)\right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di\right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

Plugging (1.1.7) makes

$$\left[C_{HF,t}(i)\right] = \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\vartheta_c} C_{HF,t}$$
(A.1.11)

(1.1.17) to (1.1.20) and (1.1.22) to (1.1.24) can, analogously, be derived.

Plugging (A.1.11) in (A.1.1) makes

$$C_{HF,t}P_{HF,t} = \int_{0}^{1} \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\vartheta_{C}} C_{HF,t}\left[P_{HF,t}(i)\right] di$$

$$C_{HF,t}P_{HF,t} = C_{HF,t} \left(\frac{1}{P_{HF,t}}\right)^{-\vartheta_c} \int_{0}^{1} \left[P_{HF,t}(i)\right]^{-\vartheta_c} \left[P_{HF,t}(i)\right] di$$

$$P_{HF,t} = \left(P_{HF,t}\right)^{\vartheta_c} \int_{0}^{1} \left[P_{HF,t}(i)\right]^{1-\vartheta_c} di$$

$$(P_{HF,t})^{1-\vartheta_c} = \int_0^1 [P_{HF,t}(i)]^{1-\vartheta_c} di$$

$$P_{HF,t} = \left[\int_{0}^{1} \left[P_{HF,t}(i) \right]^{1-\vartheta_c} di \right]^{\frac{1}{1-\vartheta_c}}$$
(A.1.12)

(1.1.11) to (1.1.12) and (1.1.14) to (1.1.16) can, analogously, be derived.

Rewriting the (1.1.2)

$$C_t P_t = B_t + L_t W_t - T_t - E_t \{ Q_{t,t+1} B_{t+1} \}$$

The left hand side of the equations shows the consumption expenditure in the period t. The right hand side term $B_t + L_t W_t - T_t$ represents the available gross income in the period t, while $E_t \{Q_{t,t+1}B_{t+1}\}$ represents the time t investment in the portfolio with the nominal payoffs B_{t+1} in the period t+1. Thus whatever income left after tax and after portfolio investment is used in consumption. The intertemporal problem for the household with respect to the optimal one period portfolio purchase writes as:

$$\underbrace{\max_{B_{t+1}}} \left[U(C_t, L_t) + E_t \beta U(C_{t+1}, L_{t+1}) \right]$$
(A.1.13)

Subject to

$$C_t P_t = B_t + L_t W_t - T_t - E_t \int V_{t,t+1} B_{t+1} d\tau$$
 (A.1.14)

$$E_t C_{t+1} P_{t+1} = E_t \left(\int \xi_{t,t+1} B_{t+1} d\tau + L_{t+1} W_{t+1} - T_{t+1} - E_t V_{t,t+2} B_{t+2} \right)$$
 (A.1.15)

 $E_t \int V_{t,t+1} B_{t+1} d\tau$ is the market price of one period portfolio yielding random payoff B_{t+1} . It has been integrated over all possible states of nature indexed by τ . $V_{t,t+1}$ is the period t price of the Arrow security¹. $\xi_{t,t+1}$ is the probability that a given state of nature is realized in the period t+1. Equivalently, the price can be written as $E_t \frac{V_{t,t+1}}{\xi_{t,t+1}} B_{t+1}$. Thus, the stochastic discount factor can be defined as:

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$$
 (A.1.16)

Rewriting (A.1.14) and (A.1.15) as:

$$C_t = \frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} d\tau$$
 (A.1.17)

$$E_t C_{t+1} = E_t \left(\int \frac{\xi_{t,t+1}}{P_{t+1}} B_{t+1} d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} - E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right)$$
 (A.1.18)

Using (1.1.1) one period utility can be given as:

$$\max_{B_{t+1}} E_0 \sum_{t=0}^{1} \beta^1 \left\{ \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu} \right\}$$

$$\underbrace{max}_{B_{t+1}} \left[\beta^0 \left\{ \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu} \right\} + E_t \beta^1 \left\{ \frac{C_{t+1}^{1-\varepsilon}}{1-\varepsilon} - \frac{L_{t+1}^{1+\nu}}{1+\nu} \right\} \right]$$

$$\underbrace{\max_{B_{t+1}}} \left[\left\{ \frac{C_t^{1-\varepsilon}}{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu} \right\} + E_t \beta \left\{ \frac{C_{t+1}^{1-\varepsilon}}{1-\varepsilon} - \frac{L_{t+1}^{1+\nu}}{1+\nu} \right\} \right]$$

Plugging the values of C_t and E_tC_{t+1} from **(A.1.17)** and **(A.1.18)** in the optimization problem.

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Arrow security is a one period security that yields one unit of domestic currency if a specific state of nature is realized in period t + 1 and zero otherwise.

$$\begin{split} \underbrace{\max_{B_{t+1}}} \left\{ & \underbrace{\left\{ \frac{\left(\frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} d\tau \right)^{1-\varepsilon}}_{1-\varepsilon} - \frac{L_t^{1+\nu}}{1+\nu} \right\} \\ & + E_t \beta \left\{ \underbrace{\left(E_t \left(\int \frac{\xi_{t,t+1}}{P_{t+1}} B_{t+1} d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} - E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right) \right)^{1-\varepsilon}}_{1-\varepsilon} \right. \\ & - \underbrace{\left. L_{t+1}^{1+\nu}}_{1+\nu} \right\} \right] \end{split}$$

First Order Condition with respect to B_{t+1}

$$\begin{split} B_{t+1} &: -\frac{1}{1-\varepsilon} \frac{1-\varepsilon}{1} \left(\frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau \right)^{1-\varepsilon-1} \frac{V_{t,t+1}}{P_t} \\ &+ \frac{1}{1-\varepsilon} \frac{1-\varepsilon}{1} \beta \left(E_t \left(\int \frac{\xi_{t,t+1}}{P_{t+1}} B_{t+1} \, d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} \right) \\ &- E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right) \right)^{1-\varepsilon-1} \frac{\xi_{t,t+1}}{P_{t+1}} = 0 \\ \\ B_{t+1} &: -\left(\frac{B_t}{P_t} + L_t \frac{W_t}{P_t} - T_t \frac{1}{P_t} - E_t \int \frac{V_{t,t+1}}{P_t} B_{t+1} \, d\tau \right)^{-\varepsilon} \frac{V_{t,t+1}}{P_t} \\ &+ \beta \left(E_t \left(\int \frac{\xi_{t,t+1}}{P_{t+1}} B_{t+1} \, d\tau + L_{t+1} \frac{W_{t+1}}{P_{t+1}} - T_{t+1} \frac{1}{P_{t+1}} \right) \\ &- E_t \frac{V_{t,t+2}}{P_{t+1}} B_{t+2} \right) \right)^{-\varepsilon} \frac{\xi_{t,t+1}}{P_{t+1}} = 0 \end{split}$$

Plugging the C_t and E_tC_{t+1} in the expression for their respective values as in **(A.1.17)** and **(A.1.18)**:

$$B_{t+1}: -(C_t)^{-\varepsilon} \frac{V_{t,t+1}}{P_t} + \beta (E_t C_{t+1})^{-\varepsilon} \frac{\xi_{t,t+1}}{P_{t+1}} = 0$$

$$B_{t+1}: (C_t)^{-\varepsilon} \frac{V_{t,t+1}}{P_t} = \beta (E_t C_{t+1})^{-\varepsilon} \frac{\xi_{t,t+1}}{P_{t+1}}$$

$$B_{t+1}: \frac{V_{t,t+1}}{\xi_{t,t+1}} = \frac{\beta (E_t C_{t+1})^{-\varepsilon}}{(C_t)^{-\varepsilon}} \frac{P_t}{P_{t+1}}$$

Plugging **(A.1.16),** $Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}}$ and solving

$$B_{t+1}: Q_{t,t+1} = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}$$

$$1 = \beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}$$
(A.1.19)

Plugging $Q_t = E_t Q_{t,t+1}$ as in Gali (2008) emerges consumption Euler equation as:

$$1 = E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\} \frac{\beta}{Q_t}$$
(A.1.20)

First Order Condition with respect to L_t

$$L_{t}: \frac{1}{1-\varepsilon} \frac{1-\varepsilon}{1} \left(\frac{B_{t}}{P_{t}} + L_{t} \frac{W_{t}}{P_{t}} - T_{t} \frac{1}{P_{t}} - E_{t} \int \frac{V_{t,t+1}}{P_{t}} B_{t+1} d\tau \right)^{1-\varepsilon-1} \frac{W_{t}}{P_{t}} - \frac{1+\nu}{1} \frac{L_{t}^{1+\nu-1}}{1+\nu}$$

$$= 0$$

$$L_{t}: \left(\frac{B_{t}}{P_{t}} + L_{t} \frac{W_{t}}{P_{t}} - T_{t} \frac{1}{P_{t}} - E_{t} \int \frac{V_{t,t+1}}{P_{t}} B_{t+1} d\tau \right)^{-\varepsilon} \frac{W_{t}}{P_{t}} - L_{t}^{v} = 0$$

Plugging the C_t in the expression for its value as in **(A.1.17)**

$$L_t: C_t^{-\varepsilon} \frac{W_t}{P_t} - L_t^{\nu} = 0$$

$$L_t: \frac{W_t}{P_t} = \frac{L_t^{\nu}}{C_t^{-\varepsilon}}$$

$$\frac{W_t}{P_t} = C_t^{\ \varepsilon} L_t^{\ \nu} \tag{A.1.21}$$

Log linearization of (A.1.20).

Given that

$$Q_t = \frac{1}{1 + i_t}$$

Taking logarithm

$$\log Q_t = \log \frac{1}{1+i_t}$$

$$\log Q_t = -\log(1+i_t)$$

$$-\log Q_t = \log(1+i_t) \simeq i_t$$

$$-\log Q_t \simeq i_t$$
 (A.1.22)

Given that

$$\beta = \frac{1}{1+r}$$

Taking logarithm

$$\log \beta = \log \frac{1}{1+\mathfrak{r}}$$

$$\log \beta = -\log(1 + r)$$

$$-\log \beta = \log(1 + r) \simeq r$$

$$-\log \beta \simeq r$$
(A.1.23)

Rewriting (A.1.20)

$$1 = E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \right\} \frac{\beta}{Q_t}$$

$$1 = E_t \left\{ e^{\log \left[\left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \right\} \frac{\beta}{Q_t} \right]} \right\}$$

$$1 = E_t \left\{ e^{\left[log \frac{P_t}{P_{t+1}} - \varepsilon log \frac{C_{t+1}}{C_t} + log \frac{\beta}{Q_t}\right]} \right\}$$

$$1 = E_t \left\{ e^{[-\{\log P_{t+1} - \log P_t\} - \varepsilon \{\log C_{t+1} - \log C_t\} + \{\log \beta - \log Q_t\}]} \right\}$$

Plugging (A.1.22) and (A.1.23)

$$1 = E_t \left\{ e^{\left[-\{p_{t+1} - p_t\} - \varepsilon \{c_{t+1} - c_t\} - r + i_t \right]} \right\}$$

Where small letter is the logarithm (with natural base) value of her corresponding capital letter and hereinafter the very same methodology is used throughout the Appendix.

$$1 = E_t \left\{ e^{\left[-\pi_{t+1} - \varepsilon \Delta c_{t+1} - r + i_t\right]} \right\}$$

Where
$$\pi_{t+1} = \{p_{t+1} - p_t\}$$
 and $\Delta c_{t+1} = c_{t+1} - c_t$

In the steady state $\mathbf{r}=i-\pi-\varepsilon\,\Delta c$ thus, the Euler equation around steady state becomes.

$$1 = E_t \left\{ e^{\left[-(\pi_{t+1} - \pi) - \varepsilon (\Delta c_{t+1} - \Delta c) - (\mathbf{r} - \mathbf{r}) + (i_t - i)\right]} \right\}$$

First order Taylor expansion yields:

$$1 = E_t \{ 1 - (\pi_{t+1} - \pi) - \varepsilon (\Delta c_{t+1} - \Delta c) - (\mathfrak{r} - \mathfrak{r}) + (i_t - i) \}$$

Plugging $\Delta c = \kappa$

$$1 = E_t \{ 1 - \pi_{t+1} + \pi - \varepsilon \Delta c_{t+1} + \varepsilon \kappa + i_t - i \}$$

$$1 = (1 + \pi - i + \varepsilon \kappa) + E_t(i_t - \pi_{t+1} - \varepsilon \Delta c_{t+1})$$

$$1 = (1 + \pi - i + \varepsilon \kappa) + (i_t - E_t \pi_{t+1} - \varepsilon E_t \Delta c_{t+1})$$

Plugging the steady state $\mathbf{r} = i - \pi - \varepsilon \Delta c = i - \pi - \varepsilon \kappa$

$$1 = (1 - r) + (i_t - E_t \pi_{t+1} - \varepsilon E_t \Delta c_{t+1})$$

Plugging $\Delta c_{t+1} = c_{t+1} - c_t$

$$0 = i_t - \mathbf{r} - E_t \pi_{t+1} - \varepsilon E_t c_{t+1} + \varepsilon c_t$$

$$c_t = E_t c_{t+1} + \frac{1}{\epsilon} (r - i_t + E_t \pi_{t+1})$$
 (A.1.24)

Taking log of **(A.1.21)** for its log linearization.

$$\log C_t^{\varepsilon} L_t^{\nu} = \log \frac{W_t}{P_t}$$

$$\varepsilon c_t + \nu l_t = w_t - p_t \tag{A.1.25}$$

Derivation of expressions/equations for 1.4 International Economic Environment

International Risk Sharing

(A.1.19) writes for generic country j, analogously, as:

$$1 = \beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\varepsilon} \frac{P_{j,t}}{P_{j,t+1}} \right\}$$

Equating **(A.1.19)** and molded **(A.1.19)** for generic country j given above as:

$$\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\} = \beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\varepsilon} \frac{P_{j,t}}{P_{j,t+1}} \right\}$$

$$1 = \frac{\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}}{\beta E_t \left\{ \frac{1}{Q_{t,t+1}} \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\varepsilon} \frac{P_{j,t}}{P_{j,t+1}} \right\}}$$

$$1 = \frac{\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}}{\beta E_t \left\{ \left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\varepsilon} \frac{P_{j,t}}{P_{j,t+1}} \right\}}$$

Plugging (1.1.47)

$$1 = \frac{\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}}{\beta E_t \left\{ \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\varepsilon} \frac{\epsilon_{j,t} P_{j,t}^j}{\epsilon_{j,t+1} P_{j,t+1}^j} \right\}}$$

$$1 = E_t \left[\frac{\left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \frac{P_t}{P_{t+1}} \right\}}{\left\{ \left(\frac{C_{t+1}^j}{C_t^j} \right)^{-\varepsilon} \frac{\epsilon_{j,t} P_{j,t}^j}{\epsilon_{j,t+1} P_{j,t+1}^j} \right\}} \right]$$

$$1 = E_t \left[\left(\frac{C_{t+1}}{C_t} \frac{C_t^j}{C_{t+1}^j} \right)^{-\epsilon} \frac{P_t}{P_{t+1}} \frac{\epsilon_{j,t+1} P_{j,t+1}^j}{\epsilon_{j,t} P_{j,t}^j} \right]$$

$$1 = E_t \left[\left(\frac{C_{t+1}}{C_t} \frac{C_t^j}{C_{t+1}^j} \right)^{-\varepsilon} \frac{\frac{\epsilon_{j,t+1} P_{j,t+1}^j}{P_{t+1}}}{\frac{\epsilon_{j,t} P_{j,t}^j}{P_t}} \right]$$

Plugging (1.1.53)

$$1 = E_t \left[\left(\frac{C_{t+1}}{C_t} \frac{C_t^j}{C_{t+1}^j} \right)^{-\varepsilon} \frac{Q_{j,t+1}}{Q_{j,t}} \right]$$

$$1 = E_t \left[\left(\frac{C_{t+1}}{C_{t+1}^j} \right)^{-\varepsilon} \left(\frac{C_t^j}{C_t} \right)^{-\varepsilon} \frac{Q_{j,t+1}}{Q_{j,t}} \right]$$

$$(C_t)^{-\varepsilon} = E_t \left[\left(\frac{C_{t+1}}{C_{t+1}^j} \right)^{-\varepsilon} \left(C_t^j \right)^{-\varepsilon} \frac{Q_{j,t+1}}{Q_{j,t}} \right]$$

$$C_t = E_t \left[\left(\frac{C_{t+1}}{C_{t+1}^j} \right) C_t^j \left(\frac{Q_{j,t+1}}{Q_{j,t}} \right)^{-\frac{1}{\varepsilon}} \right]$$

$$C_t = E_t \left[\left(\frac{C_{t+1}}{C_{t+1}^j} \right) \left(Q_{j,t+1} \right)^{-\frac{1}{\varepsilon}} \right] C_t^j \left(Q_{j,t} \right)^{\frac{1}{\varepsilon}}$$

$$C_t = \varphi_j C_t^j (Q_{j,t})^{\frac{1}{\varepsilon}}$$
 (A.1.26)

Where

$$\varphi_j = E_t \left[\left(\frac{C_{t+1}}{C_{t+1}^j} \right) (Q_{j,t+1})^{-\frac{1}{\varepsilon}} \right]$$

Derivation of expressions / equations for 1.5 Firms

Formal sector price dynamics (inflation)

Formal sector price index can be given as:

$$P_{HF,t} = \left[\theta_{HF} \left[P_{HF,t-1}\right]^{1-\vartheta_{c}} + (1-\theta_{HF}) \left[P_{HF,t}^{*}\right]^{1-\vartheta_{c}}\right]^{\frac{1}{1-\vartheta_{c}}}$$

$$P_{HF,t} \frac{1}{P_{HF,t-1}} = \left[\theta_{HF} \left[P_{HF,t-1}\right]^{1-\vartheta_{c}} + (1-\theta_{HF}) \left[P_{HF,t}^{*}\right]^{1-\vartheta_{c}}\right]^{\frac{1}{1-\vartheta_{c}}} \frac{1}{P_{HF,t-1}}$$

$$\left(\frac{P_{HF,t}}{P_{HF,t-1}}\right)^{1-\vartheta_{c}} = \left(\left[\theta_{HF} \left[P_{HF,t-1}\right]^{1-\vartheta_{c}} + (1-\theta_{HF}) \left[P_{HF,t}^{*}\right]^{1-\vartheta_{c}}\right]^{\frac{1}{1-\vartheta_{c}}} \frac{1}{P_{HF,t-1}}\right)^{1-\vartheta_{c}}$$

$$\left(\frac{P_{HF,t}}{P_{HF,t-1}}\right)^{1-\vartheta_{c}} = \left[\theta_{HF} \left[\frac{P_{HF,t-1}}{P_{HF,t-1}}\right]^{1-\vartheta_{c}} + (1-\theta_{HF}) \left[\frac{P_{HF,t}^{*}}{P_{HF,t-1}}\right]^{1-\vartheta_{c}}\right]$$

$$\left(\frac{P_{HF,t}}{P_{HF,t-1}}\right)^{1-\vartheta_{c}} = \left[\theta_{HF} + (1-\theta_{HF}) \left[\frac{P_{HF,t}}{P_{HF,t-1}}\right]^{1-\vartheta_{c}}\right]$$

Formal sector inflation is defined as:

$$\Pi_{HF,t} = \frac{P_{HF,t}}{P_{HF,t-1}}$$
 (A.2.2)

Plugging (A.2.2)

$$\left(\Pi_{HF,t}\right)^{1-\vartheta_c} = \left[\theta_{HF} + (1-\theta_{HF})\left[\frac{P_{HF,t}^*}{P_{HF,t-1}}\right]^{1-\vartheta_c}\right]$$

Linearization around steady state makes:

$$\pi_{HF,t} = (1 - \theta_{HF}) (p_{HF,t}^* - p_{HF,t-1})$$
 (A.2.3)

Optimal Price Setting

The stochastic discount factor for nominal payoffs in the period t+k is $Q_{t,t+k}$ and can be given as:

$$Q_{t,t+k} = \beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}}\right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}}\right)$$
(A.2.4)

The representative firm's profit maximization problem can be given as:

$$\underbrace{\max_{\{P_{HF,t}^*\}} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left\{ \left(P_{HF,t}^* \right) \left(Y_{HF,t+k|t}(i) \right) - \left(T C_{HF,t+k|t}^N(i) \right) \left(Y_{HF,t+k|t}(i) \right) \right\} \right]}$$
(A.2.5)

Subject to

$$\left(Y_{HF,t+k|t}(i)\right) = \left(\frac{P_{HF,t}^*}{P_{HF,t+k}}\right)^{-\vartheta_c} C_{HF,t+k}$$
(A.2.6)

Plugging **(A.2.4)** and **(A.2.6)** in **(A.2.5)** make:

$$\underbrace{\underbrace{Max}_{\{P_{HF,t}^*\}} \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}} \right) \left\{ \left(P_{HF,t}^* \right) \left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c} C_{HF,t+k} - \left(T C_{HF,t+k|t}^N(i) \right) \left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c} C_{HF,t+k} \right\} \right]$$

$$\begin{split} P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}} \right) \left\{ (1-\vartheta_c) \left[\left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c} C_{HF,t+k} \right] \right. \\ & + \left(M C_{HF,t+k}^N \right) \left[\left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c - 1} C_{HF,t+k} \right] \frac{\vartheta_c}{P_{HF,t+k}} \right\} \right] = 0 \end{split}$$

By the definition of marginal cost

$$TC_{HF,t+k|t}^{N}(i) = MC_{HF,t+k|t}^{N}$$
 (A.2.7)

Plugging (A.2.4) and (A.2.6)

$$\left(Y_{HF,t+k|t}(i)\right) = \left(\frac{P_{HF,t}^*}{P_{HF,t+k}}\right)^{-\vartheta_c} C_{HF,t+k}$$

$$\begin{split} P_{HF,t}^* : & \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left\{ (1 - \vartheta_c) \left(Y_{HF,t+k|t}(i) \right) + \left(M C_{HF,t+k|t}^N \right) \left[\left(Y_{HF,t+k|t}(i) \right) \left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-1} \right] \frac{\vartheta_c}{P_{HF,t+k}} \right\} \right] = 0 \end{split}$$

$$P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left(Y_{HF,t+k|t}(i) \right) \left\{ (1 - \vartheta_c) + \left(M C_{HF,t+k|t}^N \right) \left(\frac{\vartheta_c}{P_{HF,t}^*} \right) \right\} \right] = 0$$

$$P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left(Y_{HF,t+k|t}(i) \right) \left\{ \left(P_{HF,t}^* \right) + \left(M C_{HF,t+k|t}^N \right) \left(\frac{\vartheta_c}{1 - \vartheta_c} \right) \right\} \right] = 0$$

$$P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[Q_{t,t+k} \left(Y_{HF,t+k|t}(i) \right) \left\{ \left(P_{HF,t}^* \right) - \left(M C_{HF,t+k|t}^N \right) \left(\frac{\vartheta_c}{\vartheta_c - 1} \right) \right\} \right] = 0$$

$$\begin{split} P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \Big[Q_{t,t+k} \Big(Y_{HF,t+k|t}(i) \Big) P_{HF,t}^* \Big] \\ = \Big(\frac{\vartheta_c}{\vartheta_c - 1} \Big) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \Big[Q_{t,t+k} \Big(Y_{HF,t+k|t}(i) \Big) \big\{ M C_{HF,t+k|t}^N \big\} \Big] \end{split}$$

Again plugging **(A.2.4)** and **(A.2.6)** and solve for $P_{HF,t}^*$

$$\begin{split} &P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}} \right) \left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c} C_{HF,t+k} P_{HF,t}^* \right] \\ &= \left(\frac{\vartheta_c}{\vartheta_c - 1} \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\beta^k \left(\frac{C_{HF,t+k}}{C_{HF,t}} \right)^{-\varepsilon} \left(\frac{P_{HF,t}}{P_{HF,t+k}} \right) \left(\frac{P_{HF,t}^*}{P_{HF,t+k}} \right)^{-\vartheta_c} C_{HF,t+k} \left\{ M C_{HF,t+k}^N \right\} \right] \end{split}$$

 $P_{HF.t}^*$

$$: \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t} \left[\beta^{k} (C_{HF,t+k})^{-\varepsilon} (C_{HF,t})^{\varepsilon} (P_{HF,t}) (P_{HF,t+k})^{-1} (P_{HF,t}^{*})^{-\vartheta_{c}} (P_{HF,t+k})^{\vartheta_{c}} C_{HF,t+k} P_{HF,t}^{*} \right]$$

$$= \left(\frac{\vartheta_{c}}{\vartheta_{c}-1} \right) \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t} \left[\beta^{k} (C_{HF,t+k})^{-\varepsilon} (C_{HF,t})^{\varepsilon} (P_{HF,t}) (P_{HF,t+k})^{-1} \right]$$

$$= \left(\frac{\vartheta_{c}}{\vartheta_{c}-1} \right) \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t} \left[\beta^{k} (C_{HF,t+k})^{-\varepsilon} (C_{HF,t+k})^{\varepsilon} (P_{HF,t+k})^{\varepsilon} (P_{HF,t+k})^{-1} \right]$$

$$\begin{split} P_{HF,t}^* : \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\beta^k (C_{HF,t+k})^{1-\varepsilon} (P_{HF,t+k})^{-1} (P_{HF,t}^*)^{1-\vartheta_c} (P_{HF,t+k})^{\vartheta_c} \right] \\ = \left(\frac{\vartheta_c}{\vartheta_c - 1} \right) \sum_{k=0}^{\infty} (\theta_{HF})^k E_t \left[\frac{\beta^k (C_{HF,t+k})^{1-\varepsilon} (P_{HF,t+k})^{-1}}{(P_{HF,t}^*)^{-\vartheta_c} (P_{HF,t+k})^{\vartheta_c} \{MC_{HF,t+k}^N\}} \right] \end{split}$$

$$\begin{split} \left(P_{HF,t}^{*}\right)^{1-\vartheta_{c}} \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t} \left[\beta^{k} \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(P_{HF,t+k}\right)^{-1} \left(P_{HF,t+k}\right)^{\vartheta_{c}}\right] \\ &= \left(\frac{\vartheta_{c}}{\vartheta_{c}-1}\right) \left(P_{HF,t}^{*}\right)^{-\vartheta_{c}} \sum_{k=0}^{\infty} (\theta_{HF})^{k} E_{t} \left[\beta^{k} \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(P_{HF,t+k}\right)^{-1} \left(P_{HF,t+k}\right)^{-1} \left(P_{HF,t+k}\right)^{\vartheta_{c}} \left\{M C_{HF,t+k}^{N}\right\}^{-1}\right] \end{split}$$

$$MC_{HF,t+k|t}^{R} = \frac{MC_{HF,t+k|t}^{N}}{P_{HF,t+k}}$$
 (A.2.8)

Plugging (A.2.8)

$$\begin{split} P_{HF,t}^* \sum_{k=0}^\infty (\theta_{HF})^k E_t \left[\beta^k \left(C_{HF,t+k} \right)^{1-\varepsilon} \left(P_{HF,t+k} \right)^{\vartheta_c - 1} \right] \\ &= \left(\frac{\vartheta_c}{\vartheta_c - 1} \right) \sum_{k=0}^\infty (\theta_{HF})^k E_t \left[\beta^k \left(C_{HF,t+k} \right)^{1-\varepsilon} \left(P_{HF,t+k} \right)^{\vartheta_c} \left\{ M C_{HF,t+k|t}^R \right\} \right] \end{split}$$

 $P_{HF,t}^*$

$$= \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) \frac{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(P_{HF,t+k}\right)^{\vartheta_c} \left\{M C_{HF,t+k}^R\right\}\right]}{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(P_{HF,t+k}\right)^{\vartheta_c - 1}\right]}$$
(A.2.9)

Divide **(A.2.9)** by $P_{HF,t}$ to get the optimal real price as a weighted average of future real marginal cost.

$$\frac{P_{HF,t}^*}{P_{HF,t}}$$

$$= \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) \frac{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(\frac{P_{HF,t+k}}{P_{HF,t}}\right)^{\vartheta_c} \left\{MC_{HF,t+k}^R\right\}\right]}{E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(C_{HF,t+k}\right)^{1-\varepsilon} \left(\frac{P_{HF,t+k}}{P_{HF,t}}\right)^{\vartheta_c - 1}\right]}$$
(A.2.10)

For the flexible prices $\theta_{HF}=0$. All the firms change their prices in every period. Therefore, solve the problem for one period.

$$P_{HF,t}^* = \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) \frac{\left[\beta^0 \left(C_{HF,t}\right)^{1-\varepsilon} \left(P_{HF,t}\right)^{\vartheta_c} \left\{M C_{HF,t|t}^R\right\}\right]}{\left[\beta^0 \left(C_{HF,t}\right)^{1-\varepsilon} \left(P_{HF,t}\right)^{\vartheta_c - 1}\right]}$$

$$P_{HF,t}^* = \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) P_{HF,t} M C_{HF,t|t}^R$$

Plugging (A.2.8)

$$P_{HF,t}^* = \left(\frac{\vartheta_c}{\vartheta_c - 1}\right) M C_{HF,t|t}^N$$
(A.2.11)

This is a frictionless markup for formal sector firms.

Log linearization of **(A.2.9)** around steady state makes:

$$p_{HF,t}^* = (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(mc_{HF,t+k|t}^R - mc_{HF}^R \right) + p_{HF,t+k} \right]$$
 (A.2.12)

$$\mu_{HF} \equiv -mc_{HF}^{R} \tag{A.2.13}$$

Plugging (A.2.13) in (A.2.12) makes

$$p_{HF,t}^* = \mu_{HF} + (1 - \theta_{HF}\beta)E_t \sum_{k=0}^{\infty} (\theta_{HF})^k \left[\beta^k \left(mc_{HF,t+k|t}^R + p_{HF,t+k} \right) \right]$$
 (A.2.14)

(A.2.13) is a desired markup for the formal sector firms.

Derivation of expressions / equations for 1.6 Equilibrium Dynamics

(1.1.21), (1.1.19) and (1.1.17) make the nested demand function as under:

$$\left[C_{HF,t}(i)\right] = \gamma(1-\alpha) \left(\frac{\left[P_{HF,t}(i)\right]}{P_{HF,t}}\right)^{-\vartheta_c} \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} C_t \tag{A.3.1}$$

Plugging (A.3.1) in (1.1.7) makes:

$$C_{HF,t} = \left[\int_{0}^{1} \left[\gamma (1 - \alpha) \left(\frac{\left[P_{HF,t}(i) \right]}{P_{HF,t}} \right)^{-\vartheta_c} \left(\frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

 $C_{HF,t}$

$$= \left[\left[\gamma (1 - \alpha) \left(\frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HF,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} \int_0^1 \left[\left[P_{HF,t}(i) \right]^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

$$C_{HF,t} = \left[\left[\gamma (1 - \alpha) \left(\frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HF,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} \int_{0}^{1} \left[P_{HF,t}(i) \right]^{1 - \vartheta_c} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

(1.1.13) makes:

$$P_{HF,t}^{1-\vartheta_c} = \int_0^1 \left[P_{HF,t}(i) \right]^{1-\vartheta_c} di$$
 (A.3.2)

Inserting (A.3.2) in the expression makes:

$$C_{HF,t} = \left[\left[\gamma (1 - \alpha) \left(\frac{P_{HF,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HF,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} P_{HF,t}^{1 - \vartheta_c} \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

$$C_{HF,t} = \gamma (1 - \alpha) \left(\frac{P_{HF,t}}{P_{H,t}}\right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} C_t$$
(A.3.3)

(1.1.22), (1.1.20) and (1.1.17) make the nested demand function as under:

$$\left[C_{HI,t}(i)\right] = (1 - \gamma)(1 - \alpha) \left(\frac{\left[P_{HI,t}(i)\right]}{P_{HI,t}}\right)^{-\vartheta_c} \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} C_t \tag{A.3.4}$$

Plugging (A.3.4) in (1.1.8)

$$C_{HI,t} = \left[\int_{0}^{1} \left[(1 - \gamma)(1 - \alpha) \left(\frac{\left[P_{HI,t}(i)\right]}{P_{HI,t}} \right)^{-\vartheta_c} \left(\frac{P_{HI,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

$$\begin{split} &C_{HI,t} \\ &= \left[\left[(1-\alpha)(1 - \alpha) \left(\frac{P_{HI,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HI,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} \int_{0}^{1} \left[\left[P_{HI,t}(i) \right]^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}} \end{split}$$

$$\begin{split} C_{HI,t} &= \left[\left[(1-\alpha)(1 - \alpha) \left(\frac{P_{HI,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HI,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} \int\limits_{0}^{1} \left[P_{HI,t}(i) \right]^{1-\vartheta_c} di \, di \, dt \end{split}$$

(1.1.14) makes:

$$(P_{HI,t})^{1-\vartheta_c} = \int_0^1 [P_{HI,t}(i)]^{1-\vartheta_c} di$$
 (A.3.5)

Plugging (A.3.5) in the expression makes:

$$C_{HI,t} = \left[\left[(1 - \alpha)(1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}} \right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t} \right)^{-\vartheta_a} C_t \left(\frac{1}{P_{HI,t}} \right)^{-\vartheta_c} \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} (P_{HI,t})^{1 - \vartheta_c} \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

$$C_{HI,t} = (1 - \alpha)(1 - \gamma) \left(\frac{P_{HI,t}}{P_{H,t}}\right)^{-\vartheta_b} \left(\frac{P_{H,t}}{P_t}\right)^{-\vartheta_a} C_t$$
(A.3.6)

Demand functions for generic country j analogous to (1.1.23), (1.1.24) and (1.1.18) are given as:

$$\left[C_{H,t}^{j}(i)\right] = \left(\frac{\left[P_{H,t}(i)\right]}{P_{H,t}}\right)^{-\vartheta_{c}} C_{H,t}^{j} \tag{A.3.7}$$

$$C_{H,t}^{j} = \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}}\right)^{-\vartheta_d} C_{F,t}^{j}$$
(A.3.8)

$$C_{F,t}^{j} = \alpha \left(\frac{P_{F,t}^{j}}{P_{t}^{j}}\right)^{-v_{a}} C_{t}^{j}$$
(A.3.9)

(A.3.7), (A.3.8) and (A.3.9) make the nested demand function as:

$$\left[C_{H,t}^{j}(i)\right] = \alpha \left(\frac{\left[P_{H,t}(i)\right]}{P_{H,t}}\right)^{-\vartheta_c} \left(\frac{P_{H,t}}{\epsilon_{j,t}P_{F,t}^{j}}\right)^{-\vartheta_d} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}}\right)^{-\vartheta_a} C_{t}^{j} \tag{A.3.10}$$

(1.1.10), **(1.1.9)**, **(1.1.16)** and **(1.1.15)** become for a generic country j as:

$$C_{H,t}^{j} = \left[\int_{0}^{1} \left[C_{H,t}^{j}(i) \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$
(A.3.11)

$$C_{F,t}^{j} = \left[\int_{0}^{1} \left[C_{H,t}^{j} \right]^{\frac{\vartheta_{d}-1}{\vartheta_{d}}} dj \right]^{\frac{\vartheta_{d}}{\vartheta_{d}-1}}$$
(A.3.12)

$$P_{H,t}^{j} = \left[\int_{0}^{1} \left[P_{H,t}^{j}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
(A.3.13)

$$P_{F,t}^{j} = \left[\int_{0}^{1} \left[P_{H,t}^{j} \right]^{1-\vartheta_{d}} dj \right]^{\frac{1}{1-\vartheta_{d}}}$$
 (A.3.14)

Plugging (A.3.10) in (A.3.11) makes:

$$C_{H,t}^{j} = \left[\int_{0}^{1} \left[\alpha \left(\frac{\left[P_{H,t}(i) \right]}{P_{H,t}} \right)^{-\vartheta_{c}} \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}} \right)^{-\vartheta_{d}} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}} \right)^{-\vartheta_{a}} C_{t}^{j} \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$

$$C_{H,t}^{j} = \left[\left[\alpha \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}} \right)^{-\vartheta_{d}} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}} \right)^{-\vartheta_{a}} C_{t}^{j} \left(\frac{1}{P_{H,t}} \right)^{-\vartheta_{c}} \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} \int_{0}^{1} \left[\left[P_{H,t}(i) \right]^{-\vartheta_{c}} \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$

$$C_{H,t}^{j} = \left[\left[\alpha \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}} \right)^{-\vartheta_{d}} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}} \right)^{-\vartheta_{a}} C_{t}^{j} \left(\frac{1}{P_{H,t}} \right)^{-\vartheta_{c}} \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} \int_{0}^{1} \left[P_{H,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$

 $P_{H,t}$, analogous to **(1.1.13)**, is given as:

$$P_{H,t} = \left[\int_{0}^{1} \left[P_{H,t}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
(A.3.15)

(A.3.15) makes:

$$[P_{H,t}]^{1-\vartheta_c} = \int_0^1 [P_{H,t}(i)]^{1-\vartheta_c} di$$
 (A.3.16)

Plugging (A.3.16) in the expression makes:

$$C_{H,t}^{j} = \left[\left[\alpha \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}} \right)^{-\vartheta_{d}} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}} \right)^{-\vartheta_{a}} C_{t}^{j} \left(\frac{1}{P_{H,t}} \right)^{-\vartheta_{c}} \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} \left[P_{H,t} \right]^{1-\vartheta_{c}} \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$

$$C_{H,t}^{j} = \alpha \left(\frac{P_{H,t}}{\epsilon_{j,t} P_{F,t}^{j}} \right)^{-\vartheta_d} \left(\frac{P_{F,t}^{j}}{P_{t}^{j}} \right)^{-\vartheta_a} C_{t}^{j}$$
(A.3.17)

II Inflation Targeting Model for Indian Economy

Abstract

In this study I briefly explain the operational design of the inflation targeting framework and then derive a monetary policy rule (Taylor rule) for the Indian economy and thereby (1) to target the Indian domestic inflation and (2) to target domestic inflation with output stabilization. The reaction function of the monetary policy is derived in either of the case by minimizing the loss function of the monetary policy. Monetary policy affects the real variables of the economy in short run and in long run money becomes neutral. The Indian economy comprises of Keynesian markets in the formal sector and Classical markets in the informal sector. In this special amalgamation of Keynesian and Classical markets; study shows that when Reserve Bank of India (RBI) conducts monetary policy, the formal sector observes the fluctuations in real variables while nominal variables varies in the informal sector. In such an economic environment the study reveals that the performance of overall monetary policy is observed very poor in term of output stabilization because of this huge informal sector. Though Indian monetary authority is helpless to stabilize the real variables of the economy in the short run but at the same time it got a shiny edge, the informal sector which observes only nominal effects. The study shows that RBI got a pretty good command on price level, ceteris paribus, without affecting (or negligible effect on) the output/employment. Low and stable inflation is good for economic growth and development. How to keep the inflation low and stable? Inflation targeting framework has a solution to this issue. The study shows that RBI can efficiently control the inflation through managing general price level without making any negative impact on output/employment in short run then India should adopt inflation targeting regime to keep the inflation low and stable, which in turn good for economic growth and development. The study recommends in terms of policy that if India adopts the inflation targeting regime then the negative impacts of the inflation targeting are much less than that of the positive outcomes, therefore, India should adopt inflation targeting regime.

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Monetary Policy, Reserve Bank of India, India

It is better to be roughly right than precisely wrong.

John Maynard Keynes

1 The Art of Rowing of an Inflation Targeting Yacht

1.1 Introduction and rationale

This section tries to explain various issues related to operational design of inflation targeting; a celebrated framework which facilitates the monetary authority to attain its foremost target i.e. price stability. It is not an easy task to find many areas in macroeconomics where almost full agreement has emerged in the last few years. However, there is today a widespread and growing consensus amongst leading policy makers and academic macroeconomists that the single most important goal of monetary policy should be the pursuit of price stability, Blejer and Leone (1999). To chase the price stability central banks have recently developed a new policy tactic called inflation targeting. Inflation targeting central bank targets publically announced numerical target for annual inflation through policy instrument to stabilize inflation itself and real variables of the economy. Price stability remains prime goal of the monetary authority while other goals become subsidiary during the inflation targeting regime. To decide the monetary policy instrument many variables come into picture apart from monetary aggregates and exchange rate. To conduct inflation targeting, a high degree of transparency by publishing objectives, decisions and plans of the central bank for public is indispensable. Inflation targeting central bank has always an obligation and accountability to meet the objectives. New Zealand was the pioneer one to adopt the inflation targeting regime in 1990 and as of now i.e. May, 2013, a total of 27 industrialized and non-industrialized countries have adopted the inflation targeting regime. The inflation targeting regime has been very successful, in terms of first stabilizing the inflation and then real variables of the economy, as hitherto no country has abandoned after taking it up or even articulated any regret.

There are several issues of debate to implement the inflation targeting regime in the best possible manner; some indispensable of them are discussed below.

1.2 Price Level Targeting versus Inflation Targeting

All the central banks who have adopted inflation targeting regime have chosen to target inflation rather than price level. Which one of these two tactics would result rather better economic performance is still an open question. Indeed, it is an issue of active research and debate. Both of these two strategies are discussed below.

Price stability is often recommended as a goal for monetary policy. Price stability has been interpreted in different ways, though. Price stability can be interpreted as price level stability, that is, a stationary price level with low variance. In practice, price stability has often been interpreted as low and stable inflation, Svensson (1999). Fischer (1996) defines price stability as the stability in the average price level and it never means the low inflation. Sweden is the only country that has ever implemented price level targeting. The experiment began in September 1931 and lasted until the outbreak of World War II. Two factors led to the adoption of a price-level target. The economic factor, the sharp deflation that began in the late 1920s, the onset of the depression in 1930, and the eventual abandonment of the gold standard necessitated the mapping out of a new, coherent strategy for monetary policy, Guender and Oh (2006).

Inflation targeting is a monetary-policy strategy that was introduced in New Zealand in 1990, has been very successful in terms of stabilizing both inflation and the real economy, and as of 2007 had been adopted by more than 20 industrialized and non-industrialized countries. It is characterized by an announced numerical inflation target, an implementation of monetary policy that gives a major role to an inflation forecast and has been called 'inflation-forecast targeting', and a high degree of transparency and accountability, Svensson (2007).

1.2.1 Pros and Cons of Price Level Targeting

The price level targeting got two crucial advantages over inflation targeting. The first one is that price level targeting reduces the uncertainty about where would be the price level in long run; hence, long run economic planning is well supported in less uncertain environment. Although, McCallum (1999) has argued that the amount of long-run uncertainty about the future price level that would arise from successful adherence to an inflation target may not be all that large, it still complicates the planning process and may lead to more mistakes in investment decisions. Secondly, there is less output variance under the price level targeting than that of under inflation targeting framework. Svensson (1999) found that price-level targeting delivers a better outcome (lower variability of inflation) than inflation targeting, when the central bank acts under discretion.

Price level targeting leads to more frequently episodes of deflation and the deflation has an ability to promote financial instability through the debt-deflation mechanism which results into potentially large costs to the economy. Economy suffers more by the repeated events of financial instability caused by deflation under price level targeting than the advantage to the economy in terms of lower variability of inflation thereby the decent inconsistency in the output.

Mishkin (2000) points out problem with price-level targets that is not often mentioned in the literature is that price-level targets may make it more difficult to conduct monetary policy. With more frequent periods of deflation resulting from a price-level target, it will become more common that short-term interest rates will hit a floor of zero during deflations as occurred during the Great Depression and in Japan recently. One argument that some economists make is that when the interest rate hits a floor of zero, monetary policy becomes ineffective. Mishkin (2000) believes this argument is a fallacy for the reasons outlined in Meltzer (1995) and in Mishkin (1996a). Monetary policy works through many other asset prices besides those of short-term debt securities, and so even when short-term interest rates hit the floor of zero, monetary policy can still be effective, and indeed was so during the Great Depression Romer (1992). Nonetheless, monetary policy becomes more difficult during deflationary episodes when interest rates hit a floor of zero because the usual guides to the

conduct of monetary policy are no longer relevant. In recent years, much of the research on how central banks should optimally conduct monetary policy focus on so-called Taylor rules, in which the central bank sets the short-term interest rates at a level which depends on both output and inflation gaps. The Taylor (1999) volume is an excellent example of this type of research. However, once the interest rate hits a floor of zero, all of the research on optimal monetary policy rules represented by work of the type in the Taylor (1999) volume is no longer useful because manipulating short-term interest rates is no longer an effective tool of monetary policy. In such a deflationary environment, central banks do have the ability to lift the economy out of recession by pursuing expansionary policy and creating more liquidity, but it becomes much less clear how far they need to go. This rightfully makes central bankers quite uncomfortable. Therefore, an important disadvantage of a price-level target is therefore that it makes it more likely that deflationary environments will occur in which central bankers will be more at sea without the usual knowledge to guide them, making it harder for them to get monetary policy exactly right.

1.2.2 Preeminence of Inflation Targeting over Price Level Targeting

Inflation targeting has not only an objective to stabilize inflation around an inflation target but, in practice, also to stabilize output. In practice inflation targeting is preferred over price level targeting for a variety of advantages as discussed in Jadranka and Marina (2008): Inflation targeting regime enables monetary policy to focus on domestic considerations and to respond to shocks to the domestic economy. Inflation targeting also has the advantage that velocity shocks are largely irrelevant because the monetary policy strategy no longer relies on a stable money inflation relationship. Because an explicit numerical inflation target increases the accountability of the central bank, inflation targeting also has the potential to reduce the likelihood that the central bank will fall into the time inconsistency trap in which it tries to expand output and employment by pursuing overly expansionary monetary policy. Thus inflation targeting acts as the potential to reduce political pressures on the central bank to pursue inflationary monetary policy and thereby reduce the likelihood of time

inconsistent policymaking. The decision by monetary authorities to choose inflation targets above zero and not price level targets reflects monetary policymakers' concerns that too low an inflation can have substantial negative effects on real economic activity. There are particularly valid reasons for fearing deflation, including the possibility that it might promote financial instability and precipitate a severe economic contraction. Targeting inflation rates of above zero makes periods of deflation less likely. The evidence on inflation expectations from surveys and interest rate levels suggest that maintaining a target for inflation above zero, but not too far above, for an extended period does not lead to instability in inflation expectations or to a decline in the central bank's credibility. Another key feature of inflation-targeting regimes is that they do not ignore traditional stabilization goals. Namely, inflation targets can increase the flexibility of the central bank to respond to declines in aggregate spending, because declines in aggregate demand that cause the inflation rate to fall below the floor of the target range will automatically stimulate the central bank to loosen monetary policy without fearing that its action will trigger a rise in inflation expectations. This strategy is readily understood by the public. In general, a central bank's communications strategy means the central bank's regular procedures for communicating with the political authorities, the financial markets, and the general public. It is closely linked to the idea of transparency and has many aspects and many motivations. Aspects of communication that have been particularly emphasized by inflation-targeting central banks are the public announcement of policy objectives, open discussion of the bank's policy framework and public release of the central bank's forecast or evaluation of the economy. Inflation-targeting regimes also put great stress on making policy transparent, policy that is clear, simple and understandable, and on regular communication with the public. Channels for communication are used by central banks in inflation-targeting countries to explain the following to the general public, financial market participants and the politicians: (a) the goals and limitations of monetary policy, including the rationale for inflation targets; (b) the numerical values of the inflation targets and how they were determined; (c) how the inflation targets are to be achieved, given current economic conditions and (d) reasons for any deviations from targets. These communication efforts

have improved private sector planning by reducing uncertainty about monetary policy, interest rates and inflation. Second, they have promoted public debate of monetary policy, in part by educating the public about what a central bank can and cannot achieve. Third, they have helped clarify the responsibilities of the central bank and of politicians in the conduct of monetary policy. Transparency and communication go hand in hand with increased accountability. The strongest case of accountability of a central bank in an inflation-targeting regime is in New Zealand, where the government has the right to dismiss the Reserve Bank's governor if the inflation targets are breached.

1.3 What Measures of Inflation to Target

Mankiw and Reis (2002) argue what measure of the inflation rate inflation targeting central bank should target. Measures of the overall price level, such as the consumer price index, are widely available. Yet a price index designed to measure the cost of living is not necessarily the best one to serve as a target for a monetary authority. This issue is often implicit in discussions of monetary policy. Many economists pay close attention to core inflation defined as inflation excluding certain volatile prices, such as food and energy prices. Others suggest that commodity prices might be particularly good indicators because they are highly responsive to changing economic conditions. Similarly, during the U.S. stock market boom of the 1990s, some economists called for Fed tightening to dampen asset price inflation suggesting that the right index for monetary policy might include not only the prices of goods and services but asset prices as well. Various monetary proposals can be viewed as inflation targeting with a nonstandard price index: The gold standard uses only the price of gold, and a fixed exchange rate uses only the price of a foreign currency. Mankiw and Reis (2002) propose and explore an approach to choosing a price index for the central bank to target. They suggest the price index that, if kept on an assigned target, would lead to the greatest stability in economic activity. They called it the stability price index concept. They further argue that the key issue in the construction of any price index is the weights assigned to the prices from different sectors of the economy. When constructing a price index to measure the

cost of living, the natural weights are the share of each good in the budget of typical consumer. When constructing a price index for the monetary authority to target, additional concerns come into play: the cyclical sensitivity of each sector, the proclivity of each sector to experience idiosyncratic shocks, and the speed with which the prices in each sector respond to changing conditions.

1.3.1 What Numeric Value of Inflation to Target

A million dollar question for any central bank following inflation targeting regime is that what numerical value of inflation should be targeted? The answer is simple; the rate of inflation which can serve the economy at its best in terms of escalation of economic growth, high employment and financial stability. The price stability can pierce all the issues in a single arrow. Thus, the best value of inflation to target is the rate of inflation which can arrest the price stability. Under inflation targeting regime, the price level becomes the economy's nominal anchor, much as a monetary aggregate would be under a monetarist policy rule, Mankiw and Reis (2002). Again, in practice, price stability has often been interpreted as low and stable inflation, Svensson (1996). Low and stable inflation is good for economic growth and development for two reasons, firstly, it allows firms and households for better planning of their plans as they know that the purchasing power will not wear away swiftly and unevenly and secondly, interest rate will be lower, encouraging investment in the productivity growth. On the other side high and fluctuating inflation is devastating for the economic activities as it escalates the uncertainty of how the prices will get shaped in future. The high uncertainty of prices will, again, result into higher rate of interest. Higher rate of interest will hamper the investment and thereby devastation in economic activities and eventually, the economic down turn.

Central banks are equipped with a mixture of tools to vary the money supply in turn to capture price stability and thereby to escalate the economic growth, high employment and financial stability. So, what measures of price stability in practice are set during inflation targeting regime? Alan Greenspan has provided a widely cited definition of price stability as a rate of inflation that is sufficiently low that households and businesses do not have to take it into

account in making everyday decisions. The said definition is viable and functional and to meet this criterion the numerical value of the inflation could be 2±1 percent (in practice, inflation targeting is never strict but always flexible) for the industrialized countries and a few percent points higher for emerging market economies or developing countries.

However, inflation targeting practice deviates from theory. Theory suggests that the optimal inflation rate should be zero in the New Keynesian paradigm, or negative according to the Friedman Rule. In practice, all inflation-targeting central banks have positive targets. The deviation of inflation targeting practice from the theory is largely based on practical aftermaths.

It is argued that a positive inflation target decreases the probability of hitting the zero lower bound on nominal rates, a point that had operational importance in the global economic slowdown experienced in 2008–09.

Bernanke et al. (1999), suggest that maintaining a target for inflation above zero, but not too far above (less than 3%), for an extended period, does not lead to instability in the public's inflation expectations or to a decline in central bank credibility. Akerlof, Dickens and Perry (1996) argue that setting inflation at too low a level produces inefficiency and will result in increase the natural rate of unemployment. A more influential argument against an inflation goal of zero is that it makes it more likely that the economy will experience episodes of deflation which can be highly dangerous because it promotes financial instability and in addition can make monetary policy decisions harder. The implication is that undershooting a zero inflation target (i.e., a deflation) is potentially more costly than overshooting a zero target by the same amount. Thus, the costs of deflation are greater than the costs of inflation, so a positive inflation target is desirable to avoid the risk of deflation, and the resulting debtdeflation. The logic of this argument suggests that setting an inflation target a little above zero is worthwhile because it provides some insurance against episodes of deflation. In addition, the conventional wisdom in recent years has been that a positive inflation target is desirable if there is downward nominal wage resistance.

1.3.2 Target Horizon

The horizon for achieving the inflation target is a key element in the design of monetary policy under an inflation-targeting regime. The horizon determines the monetary policy response to shocks. The target horizon depends on the length of the transmission mechanism of monetary policy. With a longer transmission mechanism, the central bank is not able to affect inflation in the short run. Where a disinflationary strategy is employed, inflation targets are often set annually. In emerging markets, there is often a quicker pass through from policy rates to inflation, so a shorter policy horizon is warranted. And an annual target is often seen as good for accountability. Countries on a disinflationary path will often also indicate the medium-term target in order to anchor inflation expectations.

Monetary policy affects the economy and particularly inflation with long lags. In industrialized countries, lags from monetary policy to inflation are typically estimated to be on the order of two years. Shorter time horizons, such as one year, which have been common in inflation targeting regimes, can be highly problematic. The first problem with too short a horizon is that it can lead to a controllability problem: too frequent misses of the inflation target, even when monetary policy is being conducted optimally. The second problem is that it can lead to instrument instability, in which policy instruments are moved around too much in order to try to get inflation to hit its targets over the shorter horizon. A third problem is that too short a horizon implies that not enough weight is put on output fluctuations in the central bank's loss unction, Mishkin (2000).

1.4 Strict versus Flexible Target

Inflation targeting central banks have different choices of their monetary policy goals, viz., stable inflation and or stabilization of other macroeconomic variables. Monetary authority adopts strict or point target of inflation when it plays a sole game of inflation stabilization. When central bank has many more goals apart from inflation target then strict or point target of inflation is relaxed to flexible target so that inflation can move in a target range or in a target band.

Except couple of inflation targeting countries, Chile and United Kingdom, all have adopted the range of inflation target. Central bank has more explicit flexibility under range target which also conveys an important message to the public that inflation is a volatile variable and monetary authority has imperfect control over it. Range target is flawed as it raises a serious question about the ability of monetary authority of hitting the inflation target. Therefore, in practice inflation targeting central bank makes the inflation targeting range very wide and thereby reduces incredibility of policy.

1.4.1 Rules versus Discretion

Right since the inception of the central banking to date; a heated debate has been continued over whether monetary policy should be conducted in accordance with legislative rules or through the discretion of the policymaker. To conduct monetary policy, discretion is essential to offset output fluctuations in Keynesian frameworks; on the other hand, monetarists propose a tight, fixed rule to ensure price stability. Under discretion, a monetary authority is free to act in accordance with its own judgment. For example, if legislation directed the monetary authority to do its best to improve the economy's performance and gave the monetary authority the instruments that it has, the monetary authority would have a discretionary monetary policy. A rule based monetary policy restricts discretion of the policymaker. A rule involves the exercise of control over the monetary authority in such a way that restricts the monetary authority's actions which are discretionary and deviate from rule. Whether the actions of the monetary authority should be irrevocably fixed in advance by rules, laws, and unchangeable plans or whether the monetary authority should be free to act with discretion ex post with ample margin of maneuver, Alesina and Stella (2011).

1.4.2 Role of Exchange Rate

Fluctuating exchange rate makes the inflation to dance like popcorn.

Oscillation in exchange rate always has major impact on inflation in an open

economy. Depreciation of the currency, relative to one some countries, leads to rise in inflation because of pass through of higher import prices and greater demand for the country's export.

Emerging market countries, rightfully, have an even greater concern about exchange rate movements. Not only can a real appreciation make domestic industries less competitive, but it can lead to large current account deficits which might make the country more vulnerable to currency crisis if capital inflows turn to outflows. Depreciations in emerging market countries are particularly dangerous because they can trigger a financial crisis along the lines suggested in Mishkin (1996b, 1999c). These countries have much of their debt denominated in foreign currency and when the currency depreciates, this increases the debt burden of domestic firms. Since assets are typically denominated in domestic currency and so do not increase in value, there is a resulting decline in net worth. This deterioration in balance sheets then increases adverse selection and moral hazard problems, which leads to financial instability and a sharp decline in investment and economic activity. This mechanism explains why the currency crises in Mexico in 1994-95 and East Asian in 1997 pushed these countries into full-fledged financial crises which had devastating effects on their economies, Mishkin (2000).

The movements of exchange rate are one of the major concerns of the monetary policy and the central banks always put too much focus to limit the fluctuations of exchange rate. Limiting the exchange rate movement would produce exchange rate as nominal anchor of monetary policy and thereby discoursing inflation targeting. Under the inflation targeting framework hitting the inflation target should be prime goal while setting the policy instrument. There are two major issues; firstly, should inflation targeting central banks focus only on inflation target? Secondly, should they not pay any attention to limit exchange rate fluctuation? Mishkin (2000) articulates inflation targeting central banks should keep their eyes only on inflation target and they should not pay any attention to limit exchange rate fluctuation.

1.4.3 Nuts and bolts

It is turning out to be a consensus amongst the policymakers and academic macroeconomists that inflation targeting central bank must have, firstly, the inflation targeting goal above all the rest, secondly the absence of fiscal dominance and finally, independency of policy instrument. While keeping the inflation target in mind it never means that monetary authority is unconcerned to the development issues, most important is output gap but it is the responsiveness of the central bank to a shock to the economy and the way she prefers inflation stability and output gap stability over one another.

The factor that has received considerable attention in the literature on inflation targeting in emerging economies is the presence or absence of fiscal dominance. If the central bank is required to finance the government deficit by lending directly to the government or by purchasing all new issues of government bonds that the public is unwilling to purchase, it will not also be able to target the pre-announced rate of inflation. That is, if the central bank tries to use its single policy instrument to aim at two goals, one involving financing the government deficit and the other being the achievement of an inflation target, it will simply not be able to succeed in achieving both goals with the one instrument. Put more technically, if the central bank has to finance the government deficit; it will not have control over the size of its own balance sheet. Hence, it will not be able to exert a sufficient degree of influence over the policy interest rate to set in motion the effects on the transmission mechanism needed to respond to an overly high or overly low rate of inflation. Some emerging economies have dealt with this potential problem by prohibiting direct financing of government deficits by the central bank, Freedman and Robe (2010).

The condition of independency policy instrument of the monetary authority from clutches of government has been documented in the literature widely in order to facilitate the central bank to achieve her goals. Central banks have performed exceptionally well wherever they are equipped with independency of policy instrument than that of their counterparts where government interventions have been observed. The direct control of government over monetary policy actions has resulted in poor monetary policy outcomes, with a strong tendency to high rates of inflation and the use of monetary policy for political goals.

Emerging market economies, in general, suffer from some institutional weaknesses that must be taken into account to derive sound theory and policy advice. These weaknesses relate to weak fiscal regimes, risks associated with poorly regulated financial system and susceptibility to external shocks. They are suffered from structural weakness in the form of many structural bottlenecks (namely constraints occurring due to underdeveloped infrastructure such as poor irrigation and transport facilities among others). These weaknesses make the application of inflation targeting more difficult in emerging market economies as each of these problems may dominate the monetary policy and hinder the use of inflation target. This requires the careful analysis of the existing state of affairs in these economies before putting inflation target into practice so that depending upon the institutional characteristics present in the economy, the policy can be suitably amended, Mishra and Mishra (2009).

1.5 A High Degree of Transparency and Accountability of the Central Bank

It is a tradition for the central banks to keep their objectives and policy decisions confidential but inflation targeting framework demands a very high degree of transparency in terms of publication of monetary policy report, viz., plans & decisions of the policy and forecasting of nominal & real variables of the economy, on the regular basis where various analysis of monetary regime are discussed. A key aspect of such an increased communication with the public is that monetary policy actions are explained to the public thereby enhancing both the transparency of monetary policy and the degree of accountability of the central bank. However, while greater policy transparency is desirable, and indeed necessary, for an inflation targeting strategy to be effective, it is not without costs. Notably these include heightened market sensitivity to policy announcements and publications, and the difficulty of reversing transparency once attained. Furthermore, with such heightened accountability there is a danger that a deflationary bias may emerge where given the uncertainty attached to actually attaining the inflation target, central bankers may act to reach the target in advance of the specified period, which may conflict with output stabilization. The central banks need to be held accountable for their

actions in the conduct of monetary policy. The presence of preannounced inflation targets serve as a benchmark on which the performance of the monetary authorities can be evaluated, thereby increasing their accountability.

1.6 Independency of the Central Bank

Independency of the central banks simply means unleashing the monetary authority from direct political or governmental influence in the conduct of monetary policy; however, it is a multi dimension concept. Grilli et al. (1991) distinguishes between two concepts of independent central bank: political independency and economic independency but Debelle and Fischer (1994) describe them as goal independence and instrument independence. Central bank's liberty to determine goals of monetary policy is referred as goal independence central bank. When monetary policy goals are set by the government, the central bank lacks the goal independency. The instrument independence means central bank's autonomy to freely adjust its policy tools in pursuit of the goals of monetary policy without influence from the government.

The widespread agreement among the policymakers across the globe is that price stability, the ultimate goal of the monetary authority, should be entrusted to an independent central bank where both goal independence and instrument independence are required. The independent central bank is well equipped with sophisticated tools to achieve macroeconomic goals with low and stable inflation as opposed to fiscal policy through government spending often triggers higher rates of inflation.

1.7 Course of Action

Svensson (1997) provides a clear exposition of the standard inflation targeting approach. He argues that the solution to the potential problem in implementing inflation targeting consists of making the central bank's inflation forecast an explicit intermediate target. Further he says, it is a very straightforward result that hardly requires a model, he believes that it is best demonstrated with the help of a very simple model.

In a standard forward-looking open economy model, for example, in Clarida, Galí, and Gertler (1999) and Woodford (1999b, 2003), optimizing private-sector behavior is represented by two structural equations, an aggregate-supply equation (a forward-looking 'New Keynesian Phillips Curve', NKPC), an aggregate-demand equation ('Dynamic IS curve', DISC), and Taguchi Loss Function. An aggregate-supply equation, NKPC, is derived from a first-order condition for optimal price-setting by the representative supplier following Clarida, Galí, and Gertler (1999) along the lines of Calvo sticky-pricing model, Calvo (1983). Even though there are more realistic formulations, Taylor (1979, 1980) and Fischer (1977)), Calvo pricing is more comfortable, simple and gives very similar results in comparison to more complicated models. An aggregatedemand equation, DISC, is derived from the consumption Euler equation for the optimal timing of purchases following Woodford (1999a) along the lines of Dixit-Stiglitz (1977). In the model, inflation and output are both predetermined for one period, as in Bernanke and Woodford (1997), Rotemberg and Woodford (1997, 1999), and Svensson (2003), except for an unforecastable random error term that cannot be affected by monetary policy. Taguchi Loss Function (Taguchi Method), Taguchi (1986), is used to calculate the loss caused to the society for an off-target quality characteristic. Variables of the economy, viz., inflation, output gap, rate of interest (with its smoothing) and exchange rate (with its smoothing) are introduced to write the Taguchi Loss Function (monetary policy loss function). Taguchi Loss Function is minimized, subject to NKPC and DISC to get the optimal reaction (the instrument rate) of the central bank to hit the inflation target.

1.8 Concluding Remarks

A widespread consensus has been developed that price stability should be the leading goal of the monetary policy. Inflation targeting regime, one of the recently developed strategies of monetary authority, is competent enough to capture the price stability in a most efficient way with no negative effect on the economy in the long run; however, there are several issues of debate to implement this tactic in the best possible manner. Pro inflation targeting arguments stand very firmly against the price level targeting regime, another monetary policy tactic implemented only once in the history of central banking to arrest price stability. A heated debate is going on what measures of inflation to target and what magical figure of inflation to target. Under this framework the target horizon depends on the length of the transmission mechanism of monetary policy and on response of monetary policy to the shocks. When monetary authority keeps an eye on the sole objective of inflation stabilization then central bank has a point target of inflation. The point target is relaxed to flexible target so that inflation can move in a target range or in a target band while central bank has multiple goals apart from inflation stabilization. Policymakers always have some degree of discretion as oppose the toughened policy rule à la Friedman under this regime. Inflation targeting central bank closes her eyes for exchange rate fluctuation. Structural weaknesses are bottlenecks in implementation of this regime. Lastly, a very high degree of transparency, accountability and independency of central bank is very much warranted to implement the inflation targeting framework.

Which Inflation to Target, Domestic or CPI?

One of the main objectives of monetary policy is to stabilize price inflation. In a closed economy context inflation is well defined. However, in open economy two measures of inflation coexist: domestic inflation (which excludes the direct effects of exchange rate movements on domestic prices) and CPI (consumer price index) inflation (which encompasses the price movements of imported goods and services). Which one of these two measures of inflation should be targeted by the monetary authority? Movements in the exchange rate can have short-lived effects on CPI inflation. Domestic inflation, on the other hand, can be thought of as a measure of 'core' or persistent inflationary pressures by excluding the temporary effects of exchange rate movements. In practice inflation targeting central banks have adopted CPI inflation as target variable while most of the New Open Economy Macroeconomics literature suggests that monetary authority should stabilize domestic inflation as target variable.

Literature shows that trying to stabilize CPI inflation may result in higher volatility in output, interest rates and the exchange rate than targeting a measure of domestic inflation. The reason for this is that by targeting CPI inflation, monetary policy often responds to offset the inflationary effects arising from the direct exchange rate pass-through. As a result, monetary policy becomes more responsive to short-term fluctuations in inflation, leading to higher variability in interest rates, the exchange rate and output. Hence, Gali and Monacelli (2005), Clarida, Gali and Gertler (2001) and Gali (2008) suggest that targeting domestic inflation may achieve better macroeconomic outcomes (lower interest rate, exchange rate and output variability, but higher CPI inflation variability) by 'looking though' the direct exchange rate effects.

Adolfson (2001), McCallum and Nelson (2001) and Smets and Wouters (2002) on the other hand, have tended to suggest the opposite. One of the main assumptions in these studies is the speed of transmission from movements in the exchange rate into inflation. In the earlier studies, the typical assumption was that the direct pass-through happened very quickly. This meant that exchange rate movements had only temporary effects on inflation. There is little empirical evidence to support the notion of very quick direct exchange rate pass-through and as a result, the more recent studies have assumed only gradual adjustment of import prices to exchange rate fluctuations. With this assumption, exchange rate movements tend to have more gradual and persistent effects on inflation. Some studies have also modeled imports as an intermediate good used as an input into domestic production. Under this approach, exchange rate movements and import prices can influence inflation indirectly through firms' costs of production. Under these different assumptions, the research suggests that monetary policy should target CPI inflation.

Despite the apparent differing views from the literature, the common element that can be taken from the discussion is that monetary policy should focus on the measure of inflation that matters for the behavior of individuals and firms. If exchange rate movements have only short lived effects on inflation, then looking through these effects would be appropriate. If, on the other hand, exchange rate movements result in persistent effects on inflation, then responding to them makes sense.

I model domestic inflation following Gali (2008) instead of CPI inflation as there is no clear consensus in the literature on which variable of inflation is superior to target, domestic or CPI.

3 Monetary Policy Objectives in India

Low and stable inflation is good for economic growth and development but out of control inflation is the second most devastating phenomenon next to the external war for any nation, therefore, to keep the inflation low and stable i.e. (long term) price stability has become foremost goal of the monetary authority in any nation, BIS¹ (1998), Svensson (2000) and Bernanke et al. (1999).

In practice, however, central banks are responsible for a number of objectives besides price stability, such as currency stability, financial stability, growth in employment and income. The primary objectives of central banks in many cases are legally and institutionally defined. However, all objectives may not have been spelt out explicitly in the central bank legislation but may evolve through traditions and tacit understanding between the government, the central bank and other major institutions in an economy, Reddy (2002).

However, in a developing economy, like ours, the action of monetary authority is much more complex than that of her counterpart in an industrialized (developed) economy because of the supply constraints, underdeveloped financial markets and resource gap. The monetary policy has to address multiple objectives of achieving high employment level, reasonable rate of economic growth and to ensure macroeconomic stability for equitable development. But above all, monetary authority of an underdeveloped country always remains in a dilemma; what to choose from price stability and economic growth. Though the economic growth is very important and crucial factor for the inhabitants of any nation but rising prices takes the bread away from the poorest. In addition, governments in developing countries often tend to assign the monetary policy quasi fiscal responsibilities too, which include creating conditions for equitable supply of credit to various sectors in magnitude (volume) and composition

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Bank of International Settlements.

deemed fit by the government; financing government budget deficits, managing government debt, helping towards increasing exports while reducing the dependence on imports at the same time and developing, regulating, and monitoring financial institutions. Thus, the objectives of monetary policy in most developing countries often appear to be vague and unclear. In the case of India, the preamble of the Reserve Bank of India describes the basic functions of the Bank as: "... to regulate the issue of Bank Notes and keeping of reserves with a view to securing monetary stability in India and generally to operate the currency and credit system of the country to its advantage.", Singh and Kalirajan (2006). The Reserve Bank of India has enshrined the dual objective of: (1) maintaining a reasonable degree of price stability in the economy through the regulation of monetary growth and (2) ensuring adequate expansion of credit to assist economic growth, Rangrajan (1998) with the relative emphasis on these two objectives changing from time to time.

Price stability involves deciding between price level stability and low (including zero) inflation, choosing the appropriate price index, and selecting the appropriate level for a quantitative target. It also involves deciding on the role of real variables, like output, in the objectives for monetary policy. Thus, defining price stability boils down to defining the monetary policy loss function, Svensson (1999a).

4 Strict Inflation Targeting

Under the strict inflation targeting regime, central bank is committed to keep the inflation as close to a given inflation target as possible and nothing else. No (zero) weight is given to the (other) variables (nominal or real) in the loss function of the monetary policy except inflation. In this section I derive a monetary policy rule when central bank follows the strict inflation targeting framework.

Rewriting **(1.3.51)** as:

$$E_t \pi_{H,t+1} = \frac{1}{\beta} \pi_{H,t} - \frac{\rho}{\beta} \tilde{y}_t$$
 (2.2.1)

Plugging $\frac{1}{\beta} = a_1$ and $-\frac{\rho}{\beta} = a_2$ in (2.2.1)

$$E_t \pi_{H,t+1} = a_1 \pi_{H,t} + a_2 \tilde{y}_t$$

$$\pi_{H,t+1} = a_1 \pi_{H,t} + a_2 \tilde{y}_t + \eta_{t+1}$$
 (2.2.2)

Where I have used $\pi_{H,t+1} = E_t \pi_{H,t+1} + \eta_{t+1}$. Where, η_{t+1} , is an i.i.d. shock.

Rewriting (1.3.56) as:

$$E_t \tilde{y}_{t+1} = \tilde{y}_t + \frac{1}{\varepsilon_{\alpha}} i_t - \frac{1}{\varepsilon_{\alpha}} i_t^{R^n} - \frac{1}{\varepsilon_{\alpha}} E_t \pi_{H,t+1}$$
 (2.2.3)

Plugging $\frac{1}{\varepsilon_{\alpha}} = a_3$ and $-\frac{1}{\varepsilon_{\alpha}} = a_4$ in (2.2.3).

$$E_t \tilde{y}_{t+1} = \tilde{y}_t + a_3 i_t + a_4 i_t^{R^n} + a_4 E_t \pi_{H,t+1}$$

$$\tilde{y}_{t+1} = \tilde{y}_t + a_3 i_t + a_4 i_t^{R^n} + a_4 E_t \pi_{H,t+1} + \varrho_{t+1}$$
(2.2.4)

Where I used $\tilde{y}_{t+1} = E_t \tilde{y}_{t+1} + \varrho_{t+1}$. Where, ϱ_{t+1} , is an i.i.d. shock.

Rewriting (2.2.2) as:

$$\pi_{H,t+2} = a_1 E_t \pi_{H,t+1} + a_2 E_t \tilde{y}_{t+1} + \eta_{t+2}$$
 (2.2.5)

Plugging (2.2.2) and (2.2.4) in (2.2.5)

$$\pi_{H,t+2} = a_1 (a_1 \pi_{H,t} + a_2 \tilde{y}_t + \eta_{t+1}) + a_2 (\tilde{y}_t + a_3 i_t + a_4 i_t^{R^n} + a_4 E_t \pi_{H,t+1} + \varrho_{t+1}) + \eta_{t+2}$$

$$\begin{split} \pi_{H,t+2} &= a_1 a_1 \pi_{H,t} + a_1 a_2 \tilde{y}_t + a_1 \eta_{t+1} + a_2 \tilde{y}_t + a_2 a_3 i_t + a_2 a_4 i_t^{R^n} + a_2 a_4 a_1 \pi_{H,t} \\ &\quad + a_2 a_4 a_2 \tilde{y}_t + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2} \end{split}$$

$$\pi_{H,t+2} = (a_1 a_1 + a_2 a_4 a_1) \pi_{H,t} + (a_1 a_2 + a_2 + a_2 a_4 a_2) \tilde{y}_t + a_2 a_3 i_t + a_2 a_4 i_t^{R^n}$$

$$+ a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2}$$

$$\pi_{H,t+2} = b_1 \pi_{H,t} + b_2 \tilde{y}_t + b_3 i_t + b_4 i_t^{R^n} + (a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2})$$
(2.2.6)

Where I have used $(a_1a_1 + a_2a_4a_1) = b_1$, $(a_1a_2 + a_2 + a_2a_4a_2) = b_2$, $a_2a_3 = b_3$ and $a_2a_4 = b_4$

4.1 Monetary Policy Rule

Suppose monetary policy is conducted by the central bank with inflation target π^T . Interpret inflation targeting as implying that central bank's objective in period t is to choose a sequence of current and future instrument rates of monetary policy $[i_t]_{t=0}^{\infty}$ so as to minimize

$$E_t \sum_{t=0}^{\infty} \beta^t \mathcal{L}(\pi_{H,t})$$
 (2.2.7)

$$\mathcal{L}(\pi_{H,t}) = \frac{1}{2} (\pi_{H,t} - \pi^T)^2$$
 (2.2.8)

That is, central bank wishes to minimize the expected sum of discounted squared future deviations of domestic inflation from the target.

If monetary authority changes its instrument rate at time t then the instrument rate will affect the output in the t+1 time. In turn the output will affect the inflation in t+2 time, thus monetary policy affects the inflation with a longer leg than it affects the output.

Since instrument rate affects the inflation in t+2 time, therefore, inflation is expressed in $\pi_{H,t+2}$ terms in **(2.2.6)**. Instrument rate in time t will not affect the inflation in time t and in time t+1 but will affect in time t+2, t+3, t+4, ..., Instrument rate in time t+1 will only affect the inflation in time t+3, t+4, ..., The solution to the optimization problem can be found by assigning the instrument rate in time t to hit the inflation target for time t+2 on an expected basis and instrument rate in time t+1 to hit the inflation target for time t+3 and so on. Thus, central bank can find the optimal instrument rate in time t+3 as the solution to the period by period problem.

$$\underbrace{\min_{i_t} E_t \beta^2 \mathcal{L}(\pi_{H,t+2})}_{(2.2.9)}$$

Plugging (2.2.8) in (2.2.9)

$$\underbrace{\min_{i_t} E_t \beta^2 \frac{1}{2} (\pi_{H,t+2} - \pi^T)^2}$$
(2.2.10)

Plugging (2.2.6) in (2.2.10)

$$\underbrace{\min_{i_t} \beta^2 \frac{1}{2} (\left[b_1 \pi_{H,t} + b_2 \tilde{y}_t + b_3 i_t + b_4 i_t^{R^n} + (a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2}) \right]}_{-\pi^T)^2}$$

First order condition with respect to i_t makes:

$$\beta^{2} \frac{1}{2} \frac{2}{1} \frac{b_{3}}{1} \left(\left[b_{1} \pi_{H,t} + b_{2} \tilde{y}_{t} + b_{3} i_{t} + b_{4} i_{t}^{R^{n}} + \left(a_{1} \eta_{t+1} + a_{2} a_{4} \eta_{t+1} + a_{2} \varrho_{t+1} + \eta_{t+2} \right) \right] - \pi^{T} \right) = 0$$

Plugging **(2.2.6)**

$$\beta^2 \frac{b_3}{1} \left(E_t \pi_{H,t+2} - \pi^T \right) = 0$$

$$E_t \pi_{H,t+2} = \pi^T {(2.2.11)}$$

It simply means that the instrument rate should be set so that the forecast of the one year forward inflation rate from time t+1 to time t+2, conditional upon information available in time t, equals to inflation target. Thus, time t+2 inflation forecast can be considered as an explicit intermediate target. It follows that inflation targeting loss function (2.2.8) can be replaced by an intermediate loss function, the inflation targeting loss function and can be given as:

$$\mathcal{L}^{i}(E_{t}\pi_{H,t+2}) = \frac{1}{2}(E_{t}\pi_{H,t+2} - \pi^{T})^{2}$$
(2.2.12)

Instead minimizing the expected squared deviations of future inflation rate in time t+2 from the inflation target as given in the **(2.2.9)**, central bank can minimize the squared deviation of current t+2 time inflation forecast $E_t\pi_{t+2}$ from the inflation target.

$$\underbrace{\min_{i_t} \mathcal{L}^i(E_t \pi_{H,t+2})}$$
 (2.2.13)

The t+2 time inflation forecast as given in **(2.2.6)** depends only on the current state of economy, i.e. on $\pi_{H,t}$, \tilde{y}_t and on i_t , therefore, for the current state of economy **(2.2.6)** becomes as:

$$E_t \pi_{H,t+2} = b_1 \pi_{H,t} + b_2 \tilde{y}_t + b_3 i_t + b_4 i_t^{R^n}$$
(2.2.14)

And instrument rate, i_t , can be given as:

$$b_3 i_t = E_t \pi_{H,t+2} - b_1 \pi_{H,t} - b_2 \tilde{y}_t - b_4 i_t^{R^n}$$

Plugging (2.2.11)

$$b_{3}i_{t} = \pi^{T} - b_{1}\pi_{H,t} - b_{2}\tilde{y}_{t} - b_{4}i_{t}^{R^{n}}$$

$$i_{t} = \frac{1}{b_{3}}\pi^{T} - \frac{b_{1}}{b_{3}}\pi_{H,t} - \frac{b_{2}}{b_{3}}\tilde{y}_{t} - \frac{b_{4}}{b_{3}}i_{t}^{R^{n}}$$

$$i_{t} = d_{1}\pi^{T} - d_{2}\pi_{H,t} - d_{3}\tilde{y}_{t} - d_{4}i_{t}^{R^{n}}$$
(2.2.15)

Where I have used $\frac{1}{b_3} = d_1$, $\frac{b_1}{b_3} = d_2$, $\frac{b_2}{b_3} = d_3$ and $\frac{b_4}{b_3} = d_4$. **(2.2.15)** is a monetary policy rule which is identical to Taylor rule.

The instrument rate dependents on current inflation, not because current inflation targeted but because current inflation together with output predict future inflation. With this reaction function the t+2 time inflation forecast will equal the inflation target, for all values of $\pi_{H,t}$ and \tilde{y}_t . If the inflation forecast exceeds (falls shorts of) the inflation target, the instrument rate should be increased (decreased) until the inflation forecast equals the target.

The actual inflation in time t+2 will in equilibrium be given by **(2.2.6)**, **(2.2.11)** and **(2.2.14)** as:

$$\pi_{H,t+2} = E_t \pi_{H,t+2} + (a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2})$$

$$\pi_{H,t+2} = \pi^T + (a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2})$$

$$\pi_{H,t+2} - \pi^T = (a_1 \eta_{t+1} + a_2 a_4 \eta_{t+1} + a_2 \varrho_{t+1} + \eta_{t+2})$$
(2.2.16)

Thus, **(2.2.16)** shows that central bank cannot prevent deviation of inflation target caused by the i.i.d. shocks. At best it can only control the deviations of time t+2 forecast from the target. It can, therefore, be argued that the central bank should be held accountable for the forecast deviations from the target rather than the realized inflation deviations, if the forecast deviations can be observed.

5 Flexible Inflation Targeting and Output Stabilization²

When central bank has more goals apart from inflation target itself then strict or point target of inflation is relaxed to flexible target so that inflation can move in a target range or in a target band. Central bank puts weight on the other variables (nominal or real) in the loss function of the monetary policy as asked by the social planner. In this section I derive a monetary policy reaction function when central bank tries to relax inflation target in order to capture the level of output assigned by the social planner.

Consider the case when there additional stabilization goals with regards to real variables, like output. More specifically, consider the situation when there is a long run inflation target π^T but no long run output target (other than the natural rate of output), since monetary policy cannot affect the real variables of the economy in the long run. In the short run, suppose the goal of the monetary policy is to stabilize both inflation and output around the long run inflation target and natural output rate, respectively. Thus, in the goals of monetary policy, there is symmetry between inflation and output in the short run but not in the long run. This situation can be described with a period loss function:

Rewriting (2.2.2) as:

$$\pi_{H,t+1} = \pi_{H,t} + \varphi_1 \tilde{y}_t + \eta_{t+1} \tag{2.2.17}$$

Where I assume $a_1 = 1$ and used $a_2 = \varphi_1$.

Rewriting **(2.2.4)**

$$\tilde{y}_{t+1} = \tilde{y}_t + a_3 i_t + a_4 \pi_{H,t} + a_4 \phi_1 \tilde{y}_t + \varrho_{t+1}$$

Where I used, $i_t^{R^n}$, the natural rate of real interest rate to be zero.

² I borrow this section from Svensson (1997).

$$\tilde{y}_{t+1} = (1 + a_4 \phi_1) \tilde{y}_t + a_3 i_t + a_4 \pi_{H,t} + \varrho_{t+1}$$

$$\tilde{y}_{t+1} = (1 + a_4 \phi_1) \tilde{y}_t - \phi_2 i_t + \phi_2 \pi_{H,t} + \varrho_{t+1}$$

Where I used $a_3 = -\mathbf{b}_2$, $a_4 = \mathbf{b}_2$

$$\tilde{y}_{t+1} = (1 + a_4 \phi_1) \tilde{y}_t - \phi_2 (i_t - \pi_{H,t}) + \varrho_{t+1}$$

$$\tilde{y}_{t+1} = d_1 \tilde{y}_t - d_2 (i_t - \pi_{H,t}) + \varrho_{t+1}$$
 (2.2.18)

Where I used $(1 + a_4 \varphi_1) = \varphi_1$

$$E_t \sum_{t=0}^{\infty} \beta^t \mathcal{L}(\pi_{H,t}, \tilde{y}_t)$$
 (2.2.19)

$$\mathcal{L}(\pi_{H,t}, \tilde{y}_t) = \frac{1}{2} (\pi_{H,t} - \pi^T)^2 + \Im(\tilde{y}_t)^2$$
 (2.2.20)

One year control lag for inflation:

$$\mathcal{E}(\pi_{H,t}) = \min_{\widetilde{y}_t} \left\{ \frac{1}{2} \left[\left(\pi_{H,t} - \pi^T \right)^2 + \Im \widetilde{y}_t^2 \right] + \beta E_t \mathcal{E}(\pi_{H,t+1}) \right\}$$
 (2.2.21)

Subject to **(2.2.17)**

$$\pi_{H,t+1} = \pi_{H,t} + \varphi_1 \tilde{y}_t + \eta_{t+1}$$

Where output gap \tilde{y}_t is control variable. The indirect loss function is given as, $\mathcal{E}(\pi_{H,t})$ and will be quadratic function as:

$$\pounds(\pi_{H,t}) = \mathcal{H}_0 + \frac{1}{2}\mathcal{H}_1(\pi_{H,t} - \pi^T)^2$$
 (2.2.22)

Where \mathcal{H}_0 and \mathcal{H}_1 required to determined.

Rewriting **(2.2.22)** as:

$$\mathcal{E}(\pi_{H,t+1}) = \mathcal{H}_0 + \frac{1}{2}\mathcal{H}_1(\pi_{H,t+1} - \pi^T)^2$$
 (2.2.23)

Rearranging (2.2.23) as:

$$E_t \pounds \left(\pi_{H,t+1} \right) = \mathcal{H}_0 + \frac{1}{2} \mathcal{H}_1 E_t \left[\left(\pi_{H,t+1} \right)^2 + (\pi^T)^2 - 2 \left(\pi_{H,t+1} \right) (\pi^T) \right]$$

Taking partial derivation of the equation with respect to $(\pi_{H,t+1})$

$$E_t \mathcal{E}_{\pi} \left(\pi_{H,t+1} \right) = \frac{\partial \left[E_t \mathcal{E} \left(\pi_{H,t+1} \right) \right]}{\partial \pi_{H,t+1}}$$

$$= \frac{\partial \left\{ \mathcal{K}_0 + \frac{1}{2} \mathcal{K}_1 E_t \left[\left(\pi_{H,t+1} \right)^2 + (\pi^T)^2 - 2 \left(\pi_{H,t+1} \right) (\pi^T) \right] \right\}}{\partial \pi_{H,t+1}}$$

$$E_t \mathcal{E}_{\pi} \left(\pi_{H,t+1} \right) = \frac{\partial \left[E_t \mathcal{E} \left(\pi_{H,t+1} \right) \right]}{\partial \pi_{H,t+1}} = \left\{ \frac{1}{2} \mathcal{K}_1 E_t \left[2 \pi_{H,t+1} - 2 (\pi^T) \right] \right\}$$

$$E_t \mathcal{E}_{\pi} (\pi_{H,t+1}) = \frac{\partial \left[E_t \mathcal{E} (\pi_{H,t+1}) \right]}{\partial \pi_{H,t+1}} = \left\{ \mathcal{K}_1 \left[E_t \pi_{H,t+1} - \pi^T \right] \right\}$$
 (2.2.24)

First order condition of (2.2.21) is given as:

$$\ni \tilde{y}_t + \varphi_1 \beta E_t \pounds_{\pi} (\pi_{H,t+1}) = 0$$

Plugging (2.2.24)

$$\ni \tilde{y}_t + \beta \varphi_1 \mathcal{K}_1 \big[E_t \pi_{H,t+1} - \pi^T \big] = 0$$

$$\left[E_t \pi_{H,t+1} - \pi^T\right] = -\frac{\Im}{\beta \varphi_1 \mathcal{H}_1} \tilde{y}_t \tag{2.2.25}$$

$$\tilde{y}_t = -\frac{\beta \varphi_1 \mathcal{K}_1}{\Im} \left[E_t \pi_{H,t+1} - \pi^T \right]$$

Plugging $E_t \pi_{H,t+1} = \pi_{H,t} + \varphi_1 \tilde{y}_t$

$$\tilde{y}_t = -\frac{\beta \phi_1 \mathcal{K}_1}{2} \left[\pi_{H,t} + \phi_1 \tilde{y}_t - \pi^T \right]$$

$$\tilde{y}_t = -\frac{\beta \phi_1 \mathcal{K}_1}{\beta} \phi_1 \tilde{y}_t - \frac{\beta \phi_1 \mathcal{K}_1}{\beta} \left[\pi_{H,t} - \pi^T \right]$$

$$\left(1 + \frac{\beta \varphi_1^2 \mathcal{K}_1}{\Im}\right) \tilde{y}_t = -\frac{\beta \varphi_1 \mathcal{K}_1}{\Im} \left[\pi_{H,t} - \pi^T\right]$$

$$\left(\frac{\mathbf{3}+\beta \mathbf{\phi}_1^2 \mathbf{\mathcal{H}}_1}{\mathbf{3}}\right) \tilde{\mathbf{y}}_t = -\frac{\beta \mathbf{\phi}_1 \mathbf{\mathcal{H}}_1}{\mathbf{3}} \left[\pi_{H,t} - \pi^T\right]$$

$$\tilde{y}_t = -\frac{\beta \phi_1 \mathcal{K}_1}{9} \frac{9}{9 + \beta \phi_1^2 \mathcal{K}_1} \left[\pi_{H,t} - \pi^T \right]$$

$$\tilde{y}_t = -\frac{\beta \varphi_1 \mathcal{H}_1}{9 + \beta \varphi_1^2 \mathcal{H}_1} \left[\pi_{H,t} - \pi^T \right]$$
(2.2.26)

Plugging **(2.2.26)** in $E_t \pi_{H,t+1} = \pi_{H,t} + \varphi_1 \tilde{y}_t$

$$E_t \pi_{H,t+1} = \pi_{H,t} + \varphi_1 \left[-\frac{\beta \varphi_1 \mathcal{K}_1}{9 + \beta \varphi_1^2 \mathcal{K}_1} \left[\pi_{H,t} - \pi^T \right] \right]$$

$$E_{t}\pi_{H,t+1} = \pi_{H,t} + \left[-\frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}} \left[\pi_{H,t} - \pi^{T} \right] \right] + \pi^{T} - \pi^{T}$$

$$E_{t}\pi_{H,t+1} = \pi^{T} + \left[\pi_{H,t} - \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}} \left[\pi_{H,t} - \pi^{T}\right]\right] - \pi^{T}$$

$$E_{t}\pi_{H,t+1} = \pi^{T} + \left[\pi_{H,t} - \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}} \pi_{H,t} + \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}} \pi^{T}\right] - \pi^{T}$$

$$E_{t}\pi_{H,t+1} = \pi^{T} + \left[\left(1 - \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}}\right) \pi_{H,t} - \left(1 - \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}}\right) \pi^{T}\right]$$

$$E_{t}\pi_{H,t+1} = \pi^{T} + \left(1 - \frac{\beta \varphi_{1}^{2} \mathcal{K}_{1}}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}}\right) \left(\pi_{H,t} - \pi^{T}\right)$$

$$E_{t}\pi_{H,t+1} = \pi^{T} + \left(\frac{9}{9 + \beta \varphi_{1}^{2} \mathcal{K}_{1}}\right) \left(\pi_{H,t} - \pi^{T}\right)$$

$$(2.2.27)$$

Exploiting envelope theorem on (2.2.21)

$$\pounds(\pi_{H,t}) = \left\{ \frac{1}{2} \left[(\pi_{H,t} - \pi^T)^2 + \Im \tilde{y}_t^2 \right] + \beta E_t \pounds(\pi_{H,t+1}) \right\}$$

$$\pounds(\pi_{H,t}) = \frac{1}{2} (\pi_{H,t})^2 + \frac{1}{2} (\pi^T)^2 - (\pi_{H,t}\pi^T) + \frac{1}{2} \Im \tilde{y}_t^2 + \beta E_t \pounds(\pi_{H,t+1})$$

$$\pounds_{\pi}(\pi_{H,t}) = \frac{\partial}{\partial \pi_{H,t}} \pounds(\pi_{H,t})$$

$$= \frac{\partial}{\partial \pi_{H,t}} \left[\frac{1}{2} (\pi_{H,t})^2 + \frac{1}{2} (\pi^T)^2 - (\pi_{H,t}\pi^T) + \frac{1}{2} \Im \tilde{y}_t^2 + \beta E_t \pounds(\pi_{H,t+1}) \right]$$

$$\pounds_{\pi}(\pi_{H,t}) = \frac{\partial}{\partial \pi_{H,t}} \pounds(\pi_{H,t}) = \pi_{H,t} - \pi^T + \beta \mathcal{K}_1 \left[E_t \pi_{H,t+1} - \pi^T \right] = 0$$

Where I used **(2.2.24)** $E_t \pounds_{\pi}(\pi_{H,t+1}) = \{ \mathbb{K}_1[E_t \pi_{H,t+1} - \pi^T] \}$

Plugging (2.2.27) makes:

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \pi_{H,t} - \pi^{T} + \beta \mathcal{K}_{1} \left[\left[\pi^{T} + \left(\frac{9}{9 + \beta \phi_{1}^{2} \mathcal{K}_{1}} \right) (\pi_{H,t} - \pi^{T}) \right] - \pi^{T} \right]$$

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \pi_{H,t} - \pi^{T} + \beta \mathcal{K}_{1} \left(\frac{9}{9 + \beta \phi_{1}^{2} \mathcal{K}_{1}} \right) (\pi_{H,t} - \pi^{T})$$

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \left(1 + \frac{\beta \mathcal{K}_{1} 9}{9 + \beta \phi_{1}^{2} \mathcal{K}_{1}} \right) (\pi_{H,t} - \pi^{T})$$
(2.2.28)

Exploiting envelope theorem on (2.2.22)

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \frac{\partial}{\partial \pi_{H,t}} \mathcal{E}(\pi_{H,t}) = \frac{\partial}{\partial \pi_{H,t}} \left[\mathcal{K}_0 + \frac{1}{2} \mathcal{K}_1 (\pi_{H,t} - \pi^T)^2 \right]$$

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \frac{\partial}{\partial \pi_{H,t}} \mathcal{E}(\pi_{H,t}) = \mathcal{K}_1 (\pi_{H,t} - \pi^T) = 0$$

$$\mathcal{E}_{\pi}(\pi_{H,t}) = \mathcal{K}_1 (\pi_{H,t} - \pi^T)$$
(2.2.29)

By (2.2.28) and (2.2.29)

$$\begin{split} & \mathcal{K}_1 \Big(\pi_{H,t} - \pi^T \Big) = \left(1 + \frac{\beta \mathcal{K}_1 \vartheta}{\vartheta + \beta \phi_1^2 \mathcal{K}_1} \right) \Big(\pi_{H,t} - \pi^T \Big) \\ & \mathcal{K}_1 = 1 + \frac{\beta \mathcal{K}_1 \vartheta}{\vartheta + \beta \phi_1^2 \mathcal{K}_1} \\ & \beta \phi_1^2 \mathcal{K}_1^2 + \vartheta \mathcal{K}_1 - \beta \phi_1^2 \mathcal{K}_1 - \beta \mathcal{K}_1 \vartheta - \vartheta = 0 \\ & \beta \phi_1^2 \mathcal{K}_1^2 + (\vartheta - \beta \phi_1^2 - \beta \vartheta) \mathcal{K}_1 - \vartheta = 0 \\ & \mathcal{K}_1^2 + \frac{(\vartheta - \beta \phi_1^2 - \beta \vartheta)}{\beta \phi_1^2} \mathcal{K}_1 - \frac{\vartheta}{\beta \phi_1^2} = 0 \end{split}$$

$$\mathcal{K}_{1}^{2} + \left(\frac{9}{\beta \varphi_{1}^{2}} - \frac{\beta \varphi_{1}^{2}}{\beta \varphi_{1}^{2}} - \frac{\beta 9}{\beta \varphi_{1}^{2}}\right) \mathcal{K}_{1} - \frac{9}{\beta \varphi_{1}^{2}} = 0$$

$$\mathcal{K}_{1}^{2} + \left(\frac{9}{\beta \varphi_{1}^{2}} - 1 - \frac{\beta 9}{\beta \varphi_{1}^{2}}\right) \mathcal{K}_{1} - \frac{9}{\beta \varphi_{1}^{2}} = 0$$

$$\mathcal{K}_{1}^{2} + \left(\frac{9(1 - \beta)}{\beta \varphi_{1}^{2}} - 1\right) \mathcal{K}_{1} - \frac{9}{\beta \varphi_{1}^{2}} = 0$$

$$\mathcal{K}_{1}^{2} - \left(1 - \frac{9(1 - \beta)}{\beta \varphi_{1}^{2}}\right) \mathcal{K}_{1} - \frac{9}{\beta \varphi_{1}^{2}} = 0$$

Which is in the form of a quadratic equation and has a solution.

$$\mathcal{H}_1 = \frac{-\left[-\left(1-\frac{\Im(1-\beta)}{\beta\phi_1^2}\right)\right] + \sqrt{\left[-\left(1-\frac{\Im(1-\beta)}{\beta\phi_1^2}\right)\right]^2 - 4\left(-\frac{\Im}{\beta\phi_1^2}\right)}}{2}$$

$$\mathcal{H}_{1} = \frac{\left[\left(1 - \frac{\Im(1 - \beta)}{\beta \varphi_{1}^{2}}\right)\right] + \sqrt{\left[\left(1 - \frac{\Im(1 - \beta)}{\beta \varphi_{1}^{2}}\right)\right]^{2} + \left(\frac{4\Im}{\beta \varphi_{1}^{2}}\right)}}{2}$$

$$\mathcal{H}_1 = \frac{\left[\left(1 - \frac{\Im(1-\beta)}{\beta \phi_1^2}\right)\right] + \sqrt{1 - \left(\frac{2\Im(1-\beta)}{\beta \phi_1^2}\right) + \left(\frac{\Im(1-\beta)}{\beta \phi_1^2}\right)^2 + \left(\frac{4\Im}{\beta \phi_1^2}\right)}}{2}$$

$$\mathcal{H}_1$$

$$=\frac{\left[\left(1-\frac{3(1-\beta)}{\beta\phi_1^2}\right)\right]+\sqrt{1-\left(\frac{23(1-\beta)}{\beta\phi_1^2}\right)+\left(\frac{3(1-\beta)}{\beta\phi_1^2}\right)^2+\left(\frac{43}{\beta\phi_1^2}\right)-\left(\frac{43}{\phi_1^2}\right)+\left(\frac{43}{\phi_1^2}\right)}{2}$$

 \mathcal{H}_1

$$=\frac{\left[\left(1-\frac{3(1-\beta)}{\beta \phi_1^2}\right)\right]+\sqrt{1-\left(\frac{23(1-\beta)}{\beta \phi_1^2}\right)+\left(\frac{3(1-\beta)}{\beta \phi_1^2}\right)^2+\left(\frac{43}{\beta \phi_1^2}\right)-\left(\frac{43\beta}{\beta \phi_1^2}\right)+\left(\frac{43}{\phi_1^2}\right)}{2}}$$

$$\mathfrak{K}_{1} = \frac{\left[\left(1 - \frac{\Im(1-\beta)}{\beta \phi_{1}^{2}}\right)\right] + \sqrt{1 - \left(\frac{2\Im(1-\beta)}{\beta \phi_{1}^{2}}\right) + \left(\frac{\Im(1-\beta)}{\beta \phi_{1}^{2}}\right)^{2} + \left(\frac{4\Im - 4\Im \beta}{\beta \phi_{1}^{2}}\right) + \left(\frac{4\Im - 4\Im \beta}{\beta \phi_{1}^{2}}\right) + \left(\frac{4\Im - 4\Im \beta}{\beta \phi_{1}^{2}}\right)}{2}$$

Ж1

$$=\frac{\left[\left(1-\frac{9(1-\beta)}{\beta\phi_1^2}\right)\right]+\sqrt{1-\left(\frac{29(1-\beta)}{\beta\phi_1^2}\right)+\left(\frac{9(1-\beta)}{\beta\phi_1^2}\right)^2+\left(\frac{49(1-\beta)}{\beta\phi_1^2}\right)+\left(\frac{49}{\phi_1^2}\right)}}{2}$$

$$\mathcal{H}_{1} = \frac{\left[\left(1 - \frac{3(1-\beta)}{\beta \phi_{1}^{2}}\right)\right] + \sqrt{1 + \left(\frac{23(1-\beta)}{\beta \phi_{1}^{2}}\right) + \left(\frac{3(1-\beta)}{\beta \phi_{1}^{2}}\right)^{2} + \left(\frac{43}{\phi_{1}^{2}}\right)}}{2}$$

$$\mathcal{H}_{1} = \frac{\left[\left(1 - \frac{9(1 - \beta)}{\beta \varphi_{1}^{2}}\right)\right] + \sqrt{\left[1 + \left(\frac{9(1 - \beta)}{\beta \varphi_{1}^{2}}\right)\right]^{2} + \left(\frac{49}{\varphi_{1}^{2}}\right)}}{2} \ge 1$$
(2.2.30)

Rewriting **(2.2.21)** as:

$$\pounds(\pi_{H,t+1|t}) = \min_{\widetilde{y}_{t+1|t}} \left\{ \frac{1}{2} \left[\left(\pi_{H,t+1|t} - \pi^T \right)^2 + \Im \widetilde{y}_{t+1|t}^2 \right] + \beta E_t \pounds(\pi_{H,t+2|t+1}) \right\}$$
 (2.2.31)

Subject to

$$\pi_{H,t+2|t+1}=\pi_{H,t+1}+\operatorname{\varphi}_1\widetilde{y}_{t+1}$$

$$\pi_{H,t+2|t+1} = \pi_{H,t+1|t} + \eta_{t+1} + \varphi_1(\tilde{y}_{t+1|t} + \varrho_{t+1})$$

$$\pi_{H,t+2|t+1} = \pi_{H,t+1|t} + \varphi_1 \tilde{y}_{t+1|t} + (\eta_{t+1} + \varphi_1 \varrho_{t+1})$$
 (2.2.32)

Where, $\tilde{y}_{t+1|t}$, is a control variable.

Given that:

$$\tilde{y}_{t+1} = E_t \tilde{y}_{t+1} + \varrho_{t+1}$$

Plugging (2.2.18)

$$d_{1}\tilde{y}_{t} - d_{2}(i_{t} - \pi_{H,t}) + \varrho_{t+1} = E_{t}\tilde{y}_{t+1} + \varrho_{t+1}$$

$$d_{2}(i_{t} - \pi_{H,t}) = d_{1}\tilde{y}_{t} - E_{t}\tilde{y}_{t+1}$$

$$(i_{t} - \pi_{H,t}) = \frac{d_{1}}{d_{2}}\tilde{y}_{t} - \frac{1}{d_{2}}E_{t}\tilde{y}_{t+1}$$
(2.2.33)

In this case first order condition, analogous to, (2.2.25) can be given as:

$$\left[\pi_{H,t+2|t} - \pi^{T}\right] = -\frac{9}{\beta \varphi_{1} \mathcal{H}_{1}} \tilde{y}_{t+1|t}$$
 (2.2.34)

That is, t+2 time inflation forecast should equal the inflation target only if t+1 time output forecast equals the natural output rate. Else it should exceed the inflation target in proportion to how much t+1 time output forecast falls short of the natural output level. The proportionality coefficient, $\frac{3}{\beta \phi_1 \mathcal{H}_1}$, is increasing in the relative weight on output stabilization, 3, and decreasing in (short run) inflation / output trade off. Where \mathcal{H}_1 is given in the **(2.2.30)**.

Given that:

$$\pi_{H,t+2|t} = \pi_{H,t+1|t} + \varphi_1 \tilde{y}_{t+1|t}$$

$$\pi_{H,t+2|t} = \pi_{H,t} + \varphi_1 \tilde{y}_t + \varphi_1 \left[d_1 \tilde{y}_t - d_2 (i_t - \pi_{H,t}) \right]$$

$$\pi_{H,t+2|t} = \pi_{H,t} + \varphi_1 \tilde{y}_t + \varphi_1 d_1 \tilde{y}_t - \varphi_1 d_2 (i_t - \pi_{H,t})$$

$$\pi_{H,t+2|t} = \pi_{H,t} + \varphi_1 (1 + d_1) \tilde{y}_t - \varphi_1 d_2 (i_t - \pi_{H,t})$$

$$\pi_{H,t+2|t} - \pi^T = \pi_{H,t} - \pi^T + \varphi_1 (1 + d_1) \tilde{y}_t - \varphi_1 d_2 (i_t - \pi_{H,t})$$
(2.2.35)

Rewriting the reaction function (2.2.33)

$$\left(i_t - \pi_{H,t}\right) = \frac{\mathbf{d}_1}{\mathbf{d}_2} \tilde{y}_t - \frac{1}{\mathbf{d}_2} E_t \tilde{y}_{t+1}$$

Plugging **(2.2.34)** as:

$$-\frac{\beta \varphi_1 \mathcal{H}_1}{\Im} \left[\pi_{H,t+2|t} - \pi^T \right] = \tilde{y}_{t+1|t}$$

$$\left(i_t - \pi_{H,t}\right) = \frac{\mathbf{d}_1}{\mathbf{d}_2} \tilde{\mathbf{y}}_t + \frac{1}{\mathbf{d}_2} \frac{\beta \mathbf{q}_1 \mathbf{M}_1}{\Im} \left[\pi_{H,t+2|t} - \pi^T \right]$$

Plugging (2.2.35)

$$\begin{split} \left(i_{t} - \pi_{H,t}\right) &= \frac{\mathbf{d}_{1}}{\mathbf{d}_{2}} \tilde{y}_{t} + \frac{\beta \mathbf{q}_{1} \mathcal{K}_{1}}{3 \mathbf{d}_{2}} \left[\pi_{H,t} - \pi^{T} + \mathbf{q}_{1} (1 + \mathbf{d}_{1}) \tilde{y}_{t} - \mathbf{q}_{1} \mathbf{d}_{2} (i_{t} - \pi_{H,t})\right] \\ \left(i_{t} - \pi_{H,t}\right) &= \frac{\beta \mathbf{q}_{1} \mathcal{K}_{1}}{3 \mathbf{d}_{2}} \left(\pi_{H,t} - \pi^{T}\right) - \frac{\beta \mathbf{q}_{1} \mathcal{K}_{1}}{3 \mathbf{d}_{2}} \mathbf{q}_{1} \mathbf{d}_{2} (i_{t} - \pi_{H,t}) \\ &+ \left(\frac{\beta \mathbf{q}_{1} \mathcal{K}_{1} \mathbf{q}_{1}}{3 \mathbf{d}_{2}} \left[(1 + \mathbf{d}_{1})\right] + \frac{\mathbf{d}_{1}}{\mathbf{d}_{2}}\right) \tilde{y}_{t} \end{split}$$

$$\begin{aligned} \left(i_t - \pi_{H,t}\right) + \frac{\beta \mathcal{K}_1 \varphi_1^2}{\vartheta} \left(i_t - \pi_{H,t}\right) \\ &= \frac{\beta \varphi_1 \mathcal{K}_1}{\vartheta d_2} \left(\pi_{H,t} - \pi^T\right) + \left(\frac{\beta \mathcal{K}_1 \varphi_1^2}{\vartheta d_2} \left[(1 + d_1)\right] + \frac{d_1}{d_2}\right) \tilde{y}_t \end{aligned}$$

$$\left[1 + \frac{\beta \mathcal{K}_1 \phi_1^2}{\Im}\right] \left(i_t - \pi_{H,t}\right) = \frac{\beta \phi_1 \mathcal{K}_1}{\Im \phi_2} \left(\pi_{H,t} - \pi^T\right) + \left(\frac{\beta \mathcal{K}_1 \phi_1^2}{\Im \phi_2} \left[(1 + \phi_1)\right] + \frac{\phi_1}{\phi_2}\right) \tilde{y}_t$$

$$\left[\frac{\vartheta + \beta \mathcal{K}_1 \mathbf{\phi}_1^2}{\vartheta}\right] \left(i_t - \pi_{H,t}\right) = \frac{\beta \mathbf{\phi}_1 \mathcal{K}_1}{\vartheta \mathbf{d}_2} \left(\pi_{H,t} - \pi^T\right) + \left(\frac{\beta \mathcal{K}_1 \mathbf{\phi}_1^2}{\vartheta \mathbf{d}_2} \left[(1 + \mathbf{d}_1)\right] + \frac{\mathbf{d}_1}{\mathbf{d}_2}\right) \tilde{y}_t$$

$$\begin{split} \left(i_t - \pi_{H,t}\right) &= \left[\frac{\Im}{\Im + \beta \mathcal{K}_1 \phi_1^2}\right] \frac{\beta \phi_1 \mathcal{K}_1}{\Im \phi_2} \left(\pi_{H,t} - \pi^T\right) \\ &+ \left[\frac{\Im}{\Im + \beta \mathcal{K}_1 \phi_1^2}\right] \left(\frac{\beta \mathcal{K}_1 \phi_1^2}{\Im \phi_2} \left[(1 + \phi_1)\right] + \frac{\phi_1}{\phi_2}\right) \tilde{y}_t \end{split}$$

$$\begin{split} \left(i_t - \pi_{H,t}\right) &= \left[\frac{\beta \phi_1 \mathcal{K}_1}{\mathbf{d}_2 (\mathbf{3} + \beta \mathcal{K}_1 \phi_1^2)}\right] \left(\pi_{H,t} - \pi^T\right) \\ &+ \left[\frac{\mathbf{3}}{\mathbf{3} + \beta \mathcal{K}_1 \phi_1^2}\right] \left(\frac{\beta \mathcal{K}_1 \phi_1^2}{\mathbf{3} \mathbf{d}_2} + \frac{\beta \mathcal{K}_1 \phi_1^2 \mathbf{d}_1}{\mathbf{3} \mathbf{d}_2} + \frac{\mathbf{d}_1}{\mathbf{d}_2}\right) \tilde{y}_t \end{split}$$

$$\begin{split} \left(i_{t}-\pi_{H,t}\right) &= \left[\frac{\beta \phi_{1} \mathcal{K}_{1}}{\mathbf{d}_{2} \left(\mathbf{0}+\beta \mathcal{K}_{1} \phi_{1}^{2}\right)}\right] \left(\pi_{H,t}-\pi^{T}\right) \\ &+ \left(\frac{\beta \mathcal{K}_{1} \phi_{1}^{2}}{\mathbf{0} \mathbf{d}_{2}} \left[\frac{\mathbf{0}}{\mathbf{0}+\beta \mathcal{K}_{1} \phi_{1}^{2}}\right] + \frac{\beta \mathcal{K}_{1} \phi_{1}^{2} \mathbf{d}_{1}}{\mathbf{0} \mathbf{d}_{2}} \left[\frac{\mathbf{0}}{\mathbf{0}+\beta \mathcal{K}_{1} \phi_{1}^{2}}\right] \\ &+ \frac{\mathbf{d}_{1}}{\mathbf{d}_{2}} \left[\frac{\mathbf{0}}{\mathbf{0}+\beta \mathcal{K}_{1} \phi_{1}^{2}}\right] \right) \tilde{y}_{t} \end{split}$$

$$\begin{split} \left(i_t - \pi_{H,t}\right) &= \left[\frac{\beta \, \phi_1 \mathbb{M}_1}{\mathbf{d}_2 (\mathbf{i} + \beta \mathbb{M}_1 \phi_1^2)}\right] \left(\pi_{H,t} - \pi^T\right) \\ &+ \frac{1}{\mathbf{d}_2} \left(\left[\frac{\beta \mathbb{M}_1 \phi_1^2}{\mathbf{i} + \beta \mathbb{M}_1 \phi_1^2}\right] + \left[\frac{\beta \mathbb{M}_1 \phi_1^2 \mathbf{d}_1}{\mathbf{i} + \beta \mathbb{M}_1 \phi_1^2}\right] + \left[\frac{\mathbf{d}_1 \mathbf{i}}{\mathbf{i} + \beta \mathbb{M}_1 \phi_1^2}\right]\right) \tilde{y}_t \end{split}$$

$$\begin{split} \left(i_t - \pi_{H,t}\right) &= \left[\frac{\beta \phi_1 \mathcal{K}_1}{\mathbf{d}_2 (\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2)}\right] \left(\pi_{H,t} - \pi^T\right) \\ &+ \frac{1}{\mathbf{d}_2} \left(\left[\frac{\beta \mathcal{K}_1 \phi_1^2}{\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2}\right] + \left[\frac{\beta \mathcal{K}_1 \phi_1^2 \mathbf{d}_1 + \mathbf{d}_1 \mathbf{i}}{\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2}\right]\right) \tilde{y}_t \end{split}$$

$$\begin{split} \left(i_t - \pi_{H,t}\right) &= \left[\frac{\beta \phi_1 \mathcal{K}_1}{\mathbf{d}_2 (\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2)}\right] \left(\pi_{H,t} - \pi^T\right) \\ &+ \frac{1}{\mathbf{d}_2} \left(\left[\frac{\beta \mathcal{K}_1 \phi_1^2}{\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2}\right] + \mathbf{d}_1 \left[\frac{\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2}{\mathbf{i} + \beta \mathcal{K}_1 \phi_1^2}\right]\right) \tilde{y}_t \end{split}$$

$$\begin{aligned}
\left(i_{t} - \pi_{H,t}\right) &= \left[\frac{\beta \varphi_{1} \mathcal{K}_{1}}{\mathbf{d}_{2} (\mathbf{i} + \beta \mathcal{K}_{1} \varphi_{1}^{2})}\right] \left(\pi_{H,t} - \pi^{T}\right) \\
&+ \frac{1}{\mathbf{d}_{2}} \left(\left[\frac{\beta \mathcal{K}_{1} \varphi_{1}^{2}}{\mathbf{i} + \beta \mathcal{K}_{1} \varphi_{1}^{2}}\right] + \mathbf{d}_{1}\right) \tilde{y}_{t}
\end{aligned} (2.2.36)$$

(2.2.36) is the desired reaction function of the central bank when social planner asks to the monetary authority to assign a weight, \mathfrak{I} , on the output to stabilize it and monetary authority needs to accomplish dual responsibilities of targeting inflation and stabilizing output together in the short run. In the long run inflation tends to its target while output tends to its natural level.

Given that:

$$\pi_{H,t+2|t} = \pi_{H,t+1|t} + \varphi_1 \tilde{y}_{t+1|t}$$

$$\tilde{y}_{t+1|t} = \frac{1}{\varphi_1} \left(\pi_{H,t+2|t} - \pi_{H,t+1|t} \right)$$
 (2.2.37)

Plugging (2.2.37) in (2.2.34)

$$\left[\pi_{H,t+2|t} - \pi^{T}\right] = -\frac{\Im}{\beta \wp_{1} \mathcal{H}_{1}} \left[\frac{1}{\wp_{1}} \left(\pi_{H,t+2|t} - \pi_{H,t+1|t}\right)\right]$$

$$\left[\pi_{H,t+2\mid t} - \pi^T\right] = -\frac{\Im}{\beta \mathcal{K}_1 \varphi_1^2} \pi_{H,t+2\mid t} + \frac{\Im}{\beta \mathcal{K}_1 \varphi_1^2} \pi_{H,t+1\mid t}$$

$$\pi_{H,t+2|t} + \frac{3}{\beta \mathcal{K}_1 \varphi_1^2} \pi_{H,t+2|t} = \pi^T + \frac{3}{\beta \mathcal{K}_1 \varphi_1^2} \pi_{H,t+1|t}$$

$$\left(1 + \frac{\Im}{\beta \mathcal{K}_1 \mathbf{\phi}_1^2}\right) \pi_{H,t+2|t} = \pi^T + \frac{\Im}{\beta \mathcal{K}_1 \mathbf{\phi}_1^2} \pi_{H,t+1|t}$$

$$\left(\frac{\beta \mathcal{K}_1 \mathbf{\phi}_1^2 + \mathbf{b}}{\beta \mathcal{K}_1 \mathbf{\phi}_1^2}\right) \pi_{H,t+2|t} = \pi^T + \frac{\mathbf{b}}{\beta \mathcal{K}_1 \mathbf{\phi}_1^2} \pi_{H,t+1|t}$$

$$\pi_{H,t+2|t} = \frac{\beta \mathcal{K}_1 \mathbf{\varphi}_1^2}{\beta \mathcal{K}_1 \mathbf{\varphi}_1^2 + \vartheta} \pi^T + \frac{\beta \mathcal{K}_1 \mathbf{\varphi}_1^2}{\beta \mathcal{K}_1 \mathbf{\varphi}_1^2 + \vartheta} \frac{\vartheta}{\beta \mathcal{K}_1 \mathbf{\varphi}_1^2} \pi_{H,t+1|t}$$

$$\pi_{H,t+2|t} = \frac{\beta \mathcal{K}_1 \varphi_1^2}{\beta \mathcal{K}_1 \varphi_1^2 + 3} \pi^T + \frac{3}{\beta \mathcal{K}_1 \varphi_1^2 + 3} \pi_{H,t+1|t}$$
 (2.2.38)

$$\pi_{H,t+2|t} = c\pi^T + (1-c)\pi_{H,t+1|t}$$
 (2.2.39)

Where I have used, $c = \frac{\beta \mathcal{H}_1 \phi_1^2}{\beta \mathcal{H}_1 \phi_1^2 + 9}$ and $0 < c \le 1$. Thus, t + 2 time inflation forecast should equal to a weighted average of the long run inflation target and t + 1 time inflation forecast. When 9 = 0, c = 1 then the first order condition collapses to (2.2.11).

Thus, when there is some weight on output stabilization, instead of adjusting t+2 time inflation forecast all the way to the inflation target, the central bank should let it return gradually to the long run inflation target. The intuition for this is that always adjusting t+2 time inflation forecast all the way to the long run inflation target, regardless of t+1 time inflation forecast, requires more output fluctuations. If there is a positive weight on output stabilization, a gradual adjustment of t+2 time inflation forecast towards the long run inflation target reduces output fluctuations. Higher the weight on output stabilization, slower would be the adjustment of the inflation forecast

towards the long run inflation target. Both output and inflation are mean reverting, output towards the natural output level and inflation towards the inflation target.

In summary, some weight on output stabilization motivates a gradual adjustment of t+2 time inflation forecast towards the long run inflation target. The t+2 time inflation forecast is brought closer to the long run inflation target than the predetermined t+1 time inflation target, but not all the way, in order to reduce output variability. Less the weight on output stabilization, the faster would be the adjustment towards long run inflation target.

Thus, a weight on output stabilization makes inflation targeting more complicated but not overly so. The central bank has to explain that the inflation forecast is only gradually adjusted towards the long run target. The outside monitoring of the central bank needs to be somewhat more sophisticated. Inflation targeting remains intuitive and transparent.

6 Conclusion and Policy Recommendation

As monetary policy cannot affect real variables of the economy (viz., employment) in long run, except natural output/employment, Friedman (1968) and Phelps (1968) but it works well to influence the real variables in short run through sticky prices and rigid wages which explain non neutrality of money. The Indian economy comprises of two sectors, formal and informal. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector of Indian economy characterizes the complete flexibility in prices and wages and perfections in markets. Thus, Indian economy comprises of Keynesian markets in the formal sector and Classical markets in the informal sector. In this special amalgamation of Keynesian and Classical markets; when central bank conducts monetary policy, the formal sector has a real consequences (fluctuations in real variables) in the short run while informal sector has a nominal consequences (variation in the nominal prices level only). In such an economic environment, where more than 90 per cent of workforce and about 50 per cent of the national product are accounted for by the informal economy, (National Statistical Commission, Government of India, New Delhi, 2012), performance of the monetary policy is observed very poor in term of output stabilization in short run because of this huge informal sector which has pure Classical markets. Though Indian monetary authority is helpless to stabilize the real variables of the economy in short run but at the same time it got a shiny edge, the informal sector (pure Classical markets) which observes only nominal effects when monetary policy is conducted. To conduct monetary policy, monetary authority varies the instrument rate (of the monetary policy) which in turn makes the money supply to fluctuate then it (monetary policy by fluctuating the money supply) can only affect the general price level and has a very poor influence on output/employment in short run. Summarily, Reserve Bank of India got a pretty good command on price level, ceteris paribus, without affecting the output/employment (tolerable effect on output/employment).

Low and stable inflation (consistency in general price level, price stability) is good for economic growth and development. How to keep the inflation low and stable? Inflation targeting framework has a solution to this issue. The Reserve Bank of India can efficiently control the inflation through managing general price level without making any negative impact on output/employment in short run then India should adopt inflation targeting regime to keep the inflation low and stable, which in turn good for economic growth and development.

If India adopts the inflation targeting regime then the negative impacts of the inflation targeting in short run are much less than that of the positive outcomes, therefore, India should adopt inflation targeting regime.

This conclusion is based on pure theory and may have deviation from reality. This study provides opportunities for further empirical work.

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8 Appendix B

8.1 Proof of Presentation of a Paper in a Conference



Rajasthan Economic Association

30TH ANNUAL CONFERENCE JANUARY 9 - 11, 2010

• Participation Certificate •

This is to certify that <u>Girish kuman Paliwal</u>

participated in the 30th Annual Conference of Rajasthan Economic Association organized at Vardhaman Mahaveer Open University, Kota from 9th - 11th January, 2010.

He/She has also presented a paper/chaired a session on : grassroute level

democracy and NREGA

Santosh Rajpurohit

Secretary (REA)

Prof. (Dr.) M.K. Ghadoliya
Organising Secretary

8.2 Proof of Acceptance of a Manuscript for Publication



Date:28.02.2013 Hyderabad

Dear Mr. Girish Kumar Paliwal,

Greetings from IJAE Team!

This has reference to your paper titled "A Tale of Two Macro Economic Issues: Public Spending and Households Preferences", submitted for considering for the IUP Journal of Applied Economics (IJAE).

We are glad to inform you that the review team has recommended your paper for publishing in the IUP Journal of Applied Economics.

We shall keep you duly informed about the issue in which your paper will appear.

With Regards,

Dr S.V.Sri Rama Rao) Associate Editor

The IUP Publications

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8.2.1 Manuscript

A Tale of Two Macroeconomic Issues: Public Spending and Households Preferences

Girish Kumar Paliwal*

Abstract

It is a truth universally acknowledged that India is one of the most corrupt emerging economies across the globe in terms of political/bureaucratic corruption. This paper tries to explain how political/bureaucratic corruption in India affects households preferences of consumption - leisure, consumption - saving (intertemporal consumption) and consumption - demand of real balances decisions in a typical economy, dominated to informal sector, modeled in an Open Economy New Keynesian Dynamic Stochastic General Equilibrium style (NK DSGE Model) with micro-foundations. The study incorporates enormously important informal sector as the lion's share of Indian workforce works in this sector and does not keep the degree of political/bureaucratic corruption out as thriving corruption has engulfed whole of the nation. A theoretical model based on a representative household's utility function comprising of consumption, public consumption, real balances and labour supply (production) subject to its budget constraint is developed and solved. Paper shows, theoretically, that households preferences of optimal consumptionsaving decision (optimal inter-temporal consumption decision), consumption-leisure decision (optimal consumption-labour supply decision) and optimal consumption-demand of real balances decision do not have an influence of public spending on consumption, of government transfer, of political/bureaucratic corruption/embezzlement in public spending on consumption political/bureaucratic corruption/embezzlement in government transfer.

JEL Classification: D73, D11, E21, E26, E31, E41

Keywords: NK-DSGE Model, Household Utility, Informal Economy, Corruption

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A Tale of Two Macroeconomic Issues: Public Spending and Households Preferences

1. Introduction

It is a truth universally acknowledged that India is one of the most corrupt emerging economies across the globe in terms of political/bureaucratic corruption. At the same time Indian economy has relatively very large informal sector which accommodates its ninety percent workforce and contributes around half of its GDP. In such an informal economic environment this paper attempts to study the preferences of households in the presence of political/bureaucratic corruption. The related issues has been framed in an Open Economy New Keynesian Dynamic Stochastic General Equilibrium Model with micro-foundations to find out in which way the political/bureaucratic corruption has an impact on households preferences of consumption – leisure, consumption – saving (inter-temporal consumption) and consumption – demand of real balances decisions.

1.1 Corruption in India

Corruption in India is a major issue and adversely affects its economy. In 2012 India has ranked 94th out of 176 countries in Transparency International's Corruption Perceptions Index. Most of the largest sources of corruption in India are entitlement programs and social spending schemes enacted by the Indian government. Indian media has widely published allegations of corrupt Indian citizens stashing trillions of dollars in Swiss banks. India has seen many of the biggest scandals since 2010 have involved very high levels of government, including Cabinet Ministers and Chief Ministers, such as in 2G Spectrum Scam, 2010 Commonwealth Games Scam, Adarsh Housing Society Scam, Coal Mining Scam, Mining Scandal in Karnataka and Cash for Vote Scam. A November 2010 report from the Washington based Global Financial Integrity estimates that over a 60 year period, India lost US\$213 billion in illicit financial flows beginning in 1948; adjusted

for inflation, this is estimated to be 462 billion in 2010 dollars, or about \$8 billion per year (\$7 per capita per year). The report also estimated the size of India's underground economy at approximately US\$640 billion at the end of 2008 or roughly 50% of the nation's GDP. According to a 2010 The Hindu article, unofficial estimates indicate that Indians had over US\$1456 billion in black money stored in Swiss Banks (approximately USD 1.4 trillion). While some news reports claimed that data provided by the Swiss Banking Association Report (2006) showed India has more black money than the rest of the world combined, a more recent report quoted the SBA's Head of International Communications as saying that no such official Swiss Banking Association statistics exist. Another report said that Indianowned Swiss bank account assets are worth 13 times the country's national debt Wikipedia (2012).

1.2 Informal Sector in Indian Economy

The structure of emerging market economies is somewhat differ than that of advance economies due to existence of large informal sector. The structure of goods, labour and credit markets are pretty dissimilar in formal and informal sectors of the economy as agents have different endowments and constraints. In the advance economies the relative size of informal sector is much smaller to that of formal sector; therefore, it is reasonable to ignore the informal sector in advanced economies as it has negligible impact on the aggregates. But in the emerging market economies where the informal sector is relatively large and plays an important role in the economy then neglecting the informal sector would not be justified; Schneider et al. (2010). Informal sector plays a major role in employment generation, especially for the developing world; Agenor and Montiel (1996); Harris-White and Sinha (2007); Marjit and Kar (2011) and Dutta et al. (2011). The informal sector is always complex to deal with as most of the activities of this sector are gone unrecorded.

Unorganised or informal sector constitutes a pivotal part of the Indian economy. More than 90 per cent of workforce and about 50 per cent of the national product are accounted for by the informal economy. A high proportion of socially and economically underprivileged sections of society are concentrated in the informal economic activities. The high level of growth of the Indian economy during the past two decades is accompanied by increasing informalisation. There are indications of growing interlinkages between informal and formal economic activities. There has been new dynamism of the informal economy in terms of output, employment and earnings. Faster and inclusive growth needs special attention to informal economy. Sustaining high levels of growth are also intertwined with improving domestic demand of those engaged in informal economy, and addressing the needs of the sector in terms of credit, skills, technology, marketing and infrastructure (NSC, 2012).

2. Review of Literature

Batini et al. (2010) explore the costs and benefits of informality associated with the informal sector lying outside the tax regime in a two-sector New Keynesian model. The informal sector is more labour intensive, has a lower labour productivity, is untaxed and has a classical labour market. The formal sector bears all the taxation costs, produces all the government services and capital goods, and wages are determined by a real wage norm.

Batini et al. (2011) construct a two-sector, formal-informal new Keynesian closed-economy model. The informal sector is more labour intensive, is untaxed, has a classical labour market, faces high credit constraints in financing investment and is less visible in terms of observed output.

Blackburn et al. (2004) present a dynamic general equilibrium analysis of public sector corruption and economic growth. In an economy with government intervention and capital accumulation, state-appointed bureaucrats are charged with the responsibility for procuring public goods which contribute to productive

efficiency. Corruption arises because of an opportunity for bureaucrats to appropriate public funds by misinforming the government about the cost and quality of public goods provision. The incentive for each bureaucrat to do this depends on economy-wide outcomes which, in turn, depend on the behaviour of all bureaucrats.

Bridji and Charpe (2011) develop a model of an economy with dual labour markets to understand the dynamics of the informal sector over the business cycle, as well as to analyze the implication of duality for the volatility of output and the persistence of employment. The informal labour market is competitive while the formal labour market is characterized by search costs. The size of each labour market segment depends on labour demand by firms as well as participation decisions of households. Authors show that the informal sector plays the role of a buffer, expanding in periods of recessions and shrinking when recovery sets in. Authors also show that workers switching between the two labour market increases the volatility of output. Finally, labour market segmentation modifies the properties of the search model, as the competitive labour market segment reduces the volatility of employment, unless transition costs are high.

Castillo and Montoro (2009) analyze the effects of informal labor markets on the dynamics of inflation and on the transmission of aggregate demand and supply shocks. In doing so, authors incorporate the informal sector in a modified New Keynesian model with labor market frictions as in the Diamond-Mortensen-Pissarides model. Authors show that the informal economy generates a "buffer" effects that diminish the pressure of demand shocks on aggregate wages and inflation.

Cordis (2012) investigates the relation between public corruption and the composition of state government expenditures in the United States. The results suggest that the United States is not immune to the adverse effects of corruption.

Delavallade (2006) empirically examines the impact of corruption on the structure of government spending by sector. Using the three-stage least squares method on 64 countries between 1996 and 2001, author shows that public

corruption distorts the structure of public spending by reducing the portion of social expenditure (education, health and social protection) and increasing the part dedicated to public services and order, fuel and energy, culture and defense. However, civil and political rights seem to be a stronger determinant of expense on defense than corruption.

Gabriel et al. (2011) develop a closed-economy DSGE model of the Indian economy and estimate it by Bayesian Maximum Likelihood methods using Dynare. Authors build up in stages to a model with a number of features important for emerging economies in general and the Indian economy in particular: a large proportion of credit-constrained consumers, a financial accelerator facing domestic firms seeking to finance their investment and an informal sector.

Goyal (2007) represents an optimizing model of a small open emerging market economy (SOEME) with dualistic labour markets and two types of consumers, delivers a tractable model for monetary policy.

Goyal (2008) develops a simplified version of a typical dynamic stochastic open economy general equilibrium models used to analyze optimal monetary policy. Author outlines the chief modifications when dualism in labour and in consumption is introduced to adapt the model to a small open emerging market such as India. The implications of specific labour markets, and the structure of Indian inflation and its measurement are examined.

Haider et al. (2012) develop an open economy dynamic stochastic general equilibrium (DSGE) model based on New-Keynesian micro-foundations. Alongside standard features of emerging economies, such as a combination of producer and local currency pricing for exporters, foreign capital inflow in terms of foreign direct investment and oil imports. Authors also incorporate informal labor and production sectors. This customization intensifies the exposure of a developing economy to internal and external shocks in a manner consistent with the stylized facts of Business Cycle Fluctuations.

Korneliussen (2009) hints to a possible weakness of the empirical literature on corruption and government spending. Corruption affects the composition of

government spending and in particular it affects education and health spending adversely.

Mauro (1998) asks whether predatory behavior by corrupt politicians distorts the composition of government expenditure. Corruption is found to reduce government spending on education in a cross section of countries.

Monte and Papagni (2001) show that public services and goods can provide relevant inputs to private productive activities. Modern States organize the production of these inputs on the basis of taxes collected from the community. When this process is affected by bureaucrats' corruption the efficiency of public expenditure decreases. Authors deal with the long-run consequences of this form of corruption.

3. Underpinning of Theoretical Formulization of the Model

The world economy is modeled as a continuum of small open economies with identical preferences, technology, and market structure, indexed by a unit interval [0,1], so as it does not have any impact of policy decisions of any economy as in Gali and Monacelli (2005). Again, the home economy is divided in to two sectors, namely, formal and informal following Conesa, et al. (2002); Ihrig and Moe (2004); Batini et al. (2010) and Batini et al. (2011) and has together a unit mass which spreads on the interval $[0,\gamma]$, $[\gamma,1]$, respectively, moreover, both of them consume/produce continuum of differentiated goods as their population size indexed on the unit interval $[0,\gamma]$, $[\gamma,1]$, respectively.

The home economy is inhabited by an infinitely lived representative household who derives its utility from additively separable utility function comprising of consumption, public consumption, real balances and from leisure (negative utility from working/production) as $U\left(C_t, \eta G_t, \frac{M_t}{P_t}, L_t\right)$ and wishes to maximize the utility as under following Walsh (2003) and Woodford (2003).

$$Max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \eta G_t, \frac{M_t}{P_t}, L_t\right)$$
(1)

The period utility is given by as in Gali (2008).

$$Max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\varepsilon}}{1-\varepsilon} + \eta \frac{G_t^{1-\kappa}}{1-\kappa} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\tau}}{1-\tau} - \frac{L_t^{1+\nu}}{1+\nu} \right\}$$
 (2)

Where

$$U\left(C_t, \eta G_t, \frac{M_t}{P_t}, L_t\right) = \left(\frac{C_t^{1-\varepsilon}}{1-\varepsilon} + \eta \frac{G_t^{1-\kappa}}{1-\kappa} + \frac{\left(\frac{M_t}{P_t}\right)^{1-\tau}}{1-\tau} - \frac{L_t^{1+\nu}}{1+\nu}\right)$$
(3)

Subject to

$$C_t P_t + M_t + Q_t B_t = M_{t-1} + B_{t-1} + L_t W_t + \Omega_t + \eta G_t^T - T_t^d$$
(4)

Where $C_t, \eta G_t, \frac{M_t}{P_t}, L_t$ are, respectively, consumption, public consumption, real balances and labour supply indices. $U(\cdot)$ is utility function. $\varepsilon, \kappa, \tau$ and ν are intertemporal elasticities of substitution of consumption, public consumption, holding real balances and that of labour supply between periods. β is discount factor, E_0 is expectational operator, M_t is nominal money stock, P_t is general price level, B_t are domestic bonds, G_t^T is government transfer, T_t^d is distorted tax and $\Omega_t = \int_0^1 (\Omega_{i,t}) di$ is nominal profit of the firms, again $\Omega_{i,t}$ is nominal profit of the firm producing i type of good. $\eta = (1 - \zeta)$, where, ζ is a degree of political/bureaucratic corruption/embezzlement in public spending on consumption and in government transfer. $Q_t = \frac{1}{1+i_t}$, where, i_t is nominal interest rate.

$$C_t P_t = \int_0^{\gamma} [C_t^{HF}(i)] [P_t^{HF}(i)] di + \int_{\gamma}^1 [C_t^{HI}(i)] [P_t^{HI}(i)] di + \int_0^1 \int_0^1 [C_{j,t}^F(i)] [P_{j,t}^F(i)] didj$$
 (5)

$$L_t W_t = \int_0^{\gamma} [L_t^{HF}(k)] [W_t^{HF}(k)] dj + \int_{\gamma}^1 [L_t^{HI}(k)] [W_t^{HI}(k)] dj$$
 (6)

The domestic composite consumption aggregator C_t can be given following Dixit and Stiglitz (1977) as:

$$C_t = \left[(1 - \alpha)^{\frac{1}{\vartheta_a}} (C_t^H)^{\frac{\vartheta_a - 1}{\vartheta_a}} + (\alpha)^{\frac{1}{\vartheta_a}} (C_t^F)^{\frac{\vartheta_a - 1}{\vartheta_a}} \right]^{\frac{\vartheta_a}{\vartheta_a - 1}}$$
(7)

Where C_t^H and C_t^F are indices of domestic consumption of domestically and foreign produced goods, respectively and ϑ_a is intratemporal elasticity of substitution of consumption between domestically and foreign produced goods. α is degree of openness while $1-\alpha$ is home biasness. The analogous CES aggregator of domestically produced goods C_t^H can be given as:

$$C_t^H = \left[(\gamma)^{\frac{1}{\vartheta_b}} (C_t^{HF})^{\frac{\vartheta_b - 1}{\vartheta_b}} + (1 - \gamma)^{\frac{1}{\vartheta_b}} (C_t^{HI})^{\frac{\vartheta_b - 1}{\vartheta_b}} \right]^{\frac{\vartheta_b}{\vartheta_b - 1}}$$
(8)

Where C_t^{HF} and C_t^{HI} are indices of domestic consumption of domestic formal and domestic informal sectors produced goods, respectively and ϑ_b is intratemporal elasticity of substitution of consumption between formal and informal sector produced goods. γ and $1-\gamma$ are share of domestic consumption of formal and informal sector produced goods, respectively. The CES function of consumption of domestic formal sector produced goods C_t^{HF} can be given as following Woodford (2003).

$$C_t^{HF} = \left[\left(\frac{1}{\gamma} \right)^{\frac{1}{\vartheta_c}} \int\limits_0^{\gamma} \left[C_t^{HF}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(9)

Where $[C_t^{HF}(i)]$ is the quantity of good i produced in the domestic formal sector consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of formal sector produced goods. The CES function of consumption of domestic informal sector produced goods C_t^{HI} can be given as:

$$C_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right)^{\frac{1}{\vartheta_c}} \int_{\gamma}^{1} \left[C_t^{HI}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(10)

Where $[C_t^{HI}(i)]$ is the quantity of good i produced in the domestic informal sector consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of informal sector produced goods. The CES function of consumption of foreign produced goods C_t^F can be given as:

$$C_t^F = \left[\int_0^1 \left[C_{j,t}^F \right]^{\frac{\vartheta_d - 1}{\vartheta_d}} dj \right]^{\frac{\vartheta_d}{\vartheta_d - 1}} \tag{11}$$

Where $\left[C_{j,t}^F\right]$ is the index of domestic consumption of country j produced goods and ϑ_d is intratemporal elasticity of substitution of consumption of goods produced in different countries of the world. The CES function of consumption of country j produced goods $C_{i,t}^F$ can be given as:

$$C_{j,t}^{F} = \left[\int_{0}^{1} \left[C_{j,t}^{F}(i) \right]^{\frac{\vartheta_{c}-1}{\vartheta_{c}}} di \right]^{\frac{\vartheta_{c}}{\vartheta_{c}-1}}$$

$$\tag{12}$$

Where $\left[C_{j,t}^F(i)\right]$ is the quantity of good i produced in country j and consumed in period t and ϑ_c is intratemporal elasticity of substitution of consumption between varieties of country j produced goods. The corresponding consumption based price indices of (7) to (12) are given by (13) to (18), respectively as under following Benigno and Benigno (2003) and Benigno (2004).

$$P_t = \left[(1 - \alpha)(P_t^H)^{1 - \vartheta_a} + (\alpha)(P_t^F)^{1 - \vartheta_a} \right]^{\frac{1}{1 - \vartheta_a}}$$
(13)

$$P_t^H = \left[\gamma (P_t^{HF})^{1 - \vartheta_b} + (1 - \gamma) (P_t^{HI})^{1 - \vartheta_b} \right]^{\frac{1}{1 - \vartheta_b}}$$
 (14)

$$P_t^{HF} = \left[\left(\frac{1}{\gamma} \right) \int_0^{\gamma} [P_t^{HF}(i)]^{1-\vartheta_c} di \right]^{\frac{1}{1-\vartheta_c}}$$
(15)

$$P_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right) \int_{\gamma}^{1} [P_t^{HI}(i)]^{1 - \vartheta_c} di \right]^{\frac{1}{1 - \vartheta_c}}$$
 (16)

$$P_{t}^{F} = \left[\int_{0}^{1} \left[P_{j,t}^{F} \right]^{1-\vartheta_{d}} dj \right]^{\frac{1}{1-\vartheta_{d}}}$$
 (17)

$$P_{j,t}^{F} = \left[\int_{0}^{1} \left[P_{j,t}^{F}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
 (18)

Optimal allocation of goods derives the following demand functions in each category for given level of expenditure.

$$C_t^H = (1 - \alpha) \left(\frac{P_t^H}{P_t}\right)^{-\vartheta_a} C_t \tag{19}$$

$$C_t^F = (\alpha) \left(\frac{P_t^F}{P_t}\right)^{-\vartheta_a} C_t \tag{20}$$

$$C_t^{HF} = \gamma \left(\frac{P_t^{HF}}{P_t^H}\right)^{-\vartheta_b} C_t^H \tag{21}$$

$$C_t^{HI} = (1 - \gamma) \left(\frac{P_t^{HI}}{P_t^H}\right)^{-\vartheta_b} C_t^H \tag{22}$$

$$[C_t^{HF}(i)] = \left(\frac{1}{\gamma}\right) \left(\frac{[P_t^{HF}(i)]}{P_t^{HF}}\right)^{-\theta_c} C_t^{HF} \tag{23}$$

$$[C_t^{HI}(i)] = \left(\frac{1}{1-\gamma}\right) \left(\frac{[P_t^{HI}(i)]}{P_t^{HI}}\right)^{-\vartheta_c} C_t^{HI} \tag{24}$$

$$\left[C_{j,t}^{F}(i)\right] = \left(\frac{\left[P_{j,t}^{F}(i)\right]}{P_{i,t}^{F}}\right)^{-\vartheta_{c}} C_{j,t}^{F} \tag{25}$$

$$C_{j,t}^F = \left(\frac{P_{j,t}^F}{P_t^F}\right)^{-\vartheta_d} C_t^F \tag{26}$$

Economy-wide and sector-wise expenditure on consumption can be given as:

$$\int_{0}^{\gamma} [C_{t}^{HF}(i)][P_{t}^{HF}(i)]di + \int_{\gamma}^{1} [C_{t}^{HI}(i)][P_{t}^{HI}(i)]di + \int_{0}^{1} \int_{0}^{1} [C_{j,t}^{F}(i)][P_{j,t}^{F}(i)]didj = C_{t}P_{t}$$
 (27)

$$\int_{0}^{r} [C_{t}^{HF}(i)][P_{t}^{HF}(i)]di = C_{t}^{HF}P_{t}^{HF}$$
(28)

$$\int_{V}^{1} [C_t^{HI}(i)][P_t^{HI}(i)]di = C_t^{HI}P_t^{HI}$$
(29)

$$\int_{0}^{1} \int_{0}^{1} \left[C_{j,t}^{F}(i) \right] \left[P_{j,t}^{F}(i) \right] didj = C_{t}^{F} P_{t}^{F}$$
(30)

Where

$$C_t^{HF} P_t^{HF} + C_t^{HI} P_t^{HI} = C_t^H P_t^H \tag{31}$$

$$C_t^H P_t^H + C_t^F P_t^F = C_t P_t (32)$$

Households' Government counterparts of (7) to (32) are, analogously, given by (33) to (58) as under:

$$\eta G_t = \left[(1 - \alpha)^{\frac{1}{\vartheta_a}} (\eta G_t^H)^{\frac{\vartheta_a - 1}{\vartheta_a}} + (\alpha)^{\frac{1}{\vartheta_a}} (\eta G_t^F)^{\frac{\vartheta_a - 1}{\vartheta_a}} \right]^{\frac{\vartheta_a}{\vartheta_a - 1}}$$

$$= \eta \left[(1 - \alpha)^{\frac{1}{\vartheta_a}} (G_t^H)^{\frac{\vartheta_a - 1}{\vartheta_a}} + (\alpha)^{\frac{1}{\vartheta_a}} (G_t^F)^{\frac{\vartheta_a - 1}{\vartheta_a}} \right]^{\frac{\vartheta_a}{\vartheta_a - 1}}$$
(33)

$$\eta G_t^H = \left[(\gamma)^{\frac{1}{\vartheta_b}} (\eta G_t^{HF})^{\frac{\vartheta_b - 1}{\vartheta_b}} + (1 - \gamma)^{\frac{1}{\vartheta_b}} (\eta G_t^{HI})^{\frac{\vartheta_b - 1}{\vartheta_b}} \right]^{\frac{\vartheta_b}{\vartheta_b - 1}}$$

$$= \eta \left[(\gamma)^{\frac{1}{\vartheta_b}} (G_t^{HF})^{\frac{\vartheta_b - 1}{\vartheta_b}} + (1 - \gamma)^{\frac{1}{\vartheta_b}} (G_t^{HI})^{\frac{\vartheta_b - 1}{\vartheta_b}} \right]^{\frac{\vartheta_b}{\vartheta_b - 1}}$$
(34)

$$\eta G_t^{HF} = \left[\left(\frac{1}{\gamma} \right)^{\frac{1}{\vartheta_c}} \int_0^{\gamma} \left[\eta G_t^{HF}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}} = \eta \left[\left(\frac{1}{\gamma} \right)^{\frac{1}{\vartheta_c}} \int_0^{\gamma} \left[G_t^{HF}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(35)

$$\eta G_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right)^{\frac{1}{\vartheta_c}} \int_{\gamma}^{1} \left[\eta G_t^{HI}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(36)

$$= \eta \left[\left(\frac{1}{1 - \gamma} \right)^{\frac{1}{\vartheta_c}} \int\limits_{\gamma}^{1} \left[G_t^{HI}(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$

$$\eta G_t^F = \left[\int_0^1 \left[\eta G_{j,t}^F \right]^{\frac{\vartheta_d - 1}{\vartheta_d}} di \right]^{\frac{\vartheta_d}{\vartheta_d - 1}} = \eta \left[\int_0^1 \left[G_{j,t}^F \right]^{\frac{\vartheta_d - 1}{\vartheta_d}} di \right]^{\frac{\vartheta_d}{\vartheta_d - 1}}$$
(37)

$$\eta G_{j,t}^F = \left[\int_0^1 \left[\eta G_{j,t}^F(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}} = \eta \left[\int_0^1 \left[G_{j,t}^F(i) \right]^{\frac{\vartheta_c - 1}{\vartheta_c}} di \right]^{\frac{\vartheta_c}{\vartheta_c - 1}}$$
(38)

$$P_t = \left[(1 - \alpha)(P_t^H)^{1 - \vartheta_a} + (\alpha)(P_t^F)^{1 - \vartheta_a} \right]^{\frac{1}{1 - \vartheta_a}}$$
(39)

$$P_t^H = \left[\gamma (P_t^{HF})^{1 - \vartheta_b} + (1 - \gamma) (P_t^{HI})^{1 - \vartheta_b} \right]^{\frac{1}{1 - \vartheta_b}}$$
(40)

$$P_t^{HF} = \left[\left(\frac{1}{\gamma} \right) \int_0^{\gamma} [P_t^{HF}(i)]^{1 - \vartheta_c} di \right]^{\frac{1}{1 - \vartheta_c}}$$

$$\tag{41}$$

$$P_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right) \int_{\gamma}^{1} [P_t^{HI}(i)]^{1 - \vartheta_c} di \right]^{\frac{1}{1 - \vartheta_c}}$$
(42)

$$P_t^F = \left[\int_0^1 \left[P_{j,t}^F \right]^{1-\vartheta_d} di \right]^{\frac{1}{1-\vartheta_d}} \tag{43}$$

$$P_{j,t}^{F} = \left[\int_{0}^{1} \left[P_{j,t}^{F}(i) \right]^{1-\vartheta_{c}} di \right]^{\frac{1}{1-\vartheta_{c}}}$$
(44)

$$\eta G_t^H = \eta (1 - \alpha) \left(\frac{P_t^H}{P_t}\right)^{-\vartheta_a} G_t \tag{45}$$

$$\eta G_t^F = \eta(\alpha) \left(\frac{P_t^F}{P_t}\right)^{-\vartheta_a} G_t \tag{46}$$

$$\eta G_t^{HF} = \eta \gamma \left(\frac{P_t^{HF}}{P_t^H}\right)^{-\vartheta_b} G_t^H \tag{47}$$

$$\eta G_t^{HI} = \eta (1 - \gamma) \left(\frac{P_t^{HI}}{P_t^H} \right)^{-\vartheta_b} G_t^H \tag{48}$$

$$\eta[G_t^{HF}(i)] = \eta\left(\frac{1}{\gamma}\right) \left(\frac{[P_t^{HF}(i)]}{P_t^{HF}}\right)^{-\vartheta_c} G_t^{HF} \tag{49}$$

$$\eta[G_t^{HI}(i)] = \eta\left(\frac{1}{1-\gamma}\right) \left(\frac{[P_t^{HI}(i)]}{P_t^{HI}}\right)^{-\vartheta_c} G_t^{HI}$$
(50)

$$\eta \left[G_{j,t}^F(i) \right] = \eta \left(\frac{\left[P_{j,t}^F(i) \right]}{P_{j,t}^F} \right)^{-\vartheta_c} G_{j,t}^F \tag{51}$$

$$\eta G_{j,t}^F = \eta \left(\frac{P_{j,t}^F}{P_t^F}\right)^{-\vartheta_d} G_t^F \tag{52}$$

Economy-wide and sector-wise public spending on consumption can be given as:

$$\eta \int_{0}^{\gamma} [G_{t}^{HF}(i)][P_{t}^{HF}(i)]di
+ \eta \int_{\gamma}^{1} [G_{t}^{HI}(i)][P_{t}^{HI}(i)]di + \eta \int_{0}^{1} \int_{0}^{1} [G_{j,t}^{F}(i)][P_{j,t}^{F}(i)]didj = \eta G_{t}P_{t}$$
(53)

$$\eta \int_{0}^{\gamma} [G_{t}^{HF}(i)][P_{t}^{HF}(i)]di = \eta G_{t}^{HF}P_{t}^{HF}$$
(54)

$$\eta \int_{\gamma}^{1} [G_{t}^{HI}(i)][P_{t}^{HI}(i)]di = \eta G_{t}^{HI}P_{t}^{HI}$$
(55)

$$\eta \int_{0}^{1} \int_{0}^{1} \left[G_{j,t}^{F}(i) \right] \left[P_{j,t}^{F}(i) \right] didj = \eta G_{t}^{F} P_{t}^{F}$$
(56)

Where

$$\eta G_t^{HF} P_t^{HF} + \eta G_t^{HI} P_t^{HI} = \eta G_t^H P_t^H \tag{57}$$

$$\eta G_t^H P_t^H + \eta G_t^F P_t^F = \eta G_t P_t \tag{58}$$

The domestic labour supply aggregator L_t , analogous to (8) can be given following Dixit and Stiglitz (1977) as:

$$L_{t} = \left[(\gamma)^{\frac{1}{\vartheta_{e}}} (L_{t}^{HF})^{\frac{\vartheta_{e}-1}{\vartheta_{e}}} + (1-\gamma)^{\frac{1}{\vartheta_{e}}} (L_{t}^{HI})^{\frac{\vartheta_{e}-1}{\vartheta_{e}}} \right]^{\frac{\vartheta_{e}}{\vartheta_{e}-1}}$$

$$(59)$$

Where L_t^{HF} and L_t^{HI} are indices of domestic labour supply in domestic formal and informal sectors, respectively and ϑ_e is intratemporal elasticity of substitution of labour supply between formal and informal sectors. γ and $1-\gamma$ are share of domestic labour supply in formal and informal sectors, respectively. The CES function of labour supply in domestic formal sector L_t^{HF} can be given as under:

$$L_t^{HF} = \left[\left(\frac{1}{\gamma} \right)^{\frac{1}{\vartheta_f}} \int_0^{\gamma} \left[L_t^{HF}(k) \right]^{\frac{\vartheta_f - 1}{\vartheta_f}} di \right]^{\frac{\vartheta_f}{\vartheta_f - 1}}$$
(60)

Where $[L_t^{HF}(k)]$ is the quantity of type k labour supplied in domestic formal sector in period t and ϑ_f is intratemporal elasticity of substitution between varieties of labour supplied to formal sector. The CES function of labour supply in domestic informal sector L_t^{HI} can be given as:

$$L_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right)^{\frac{1}{\vartheta_f}} \int_{\gamma}^{1} \left[L_t^{HI}(k) \right]^{\frac{\vartheta_f - 1}{\vartheta_f}} di \right]^{\frac{\vartheta_f}{\vartheta_f - 1}}$$
(61)

Where $[L_t^{HI}(k)]$ is the quantity of type k labour supplied in domestic informal sector in period t and ϑ_f is intratemporal elasticity of substitution between varieties of labour supplied to informal sector. The corresponding labour supply based wage indices of (59) to (61) are given by (62) to (64), respectively as under:

$$W_{t} = \left[\gamma(W_{t}^{HF})^{1-\vartheta_{e}} + (1-\gamma)(W_{t}^{HI})^{1-\vartheta_{e}} \right]^{\frac{1}{1-\vartheta_{e}}}$$
(62)

$$W_t^{HF} = \left[\left(\frac{1}{\gamma} \right) \int_0^{\gamma} [W_t^{HF}(k)]^{1-\vartheta_f} di \right]^{\frac{1}{1-\vartheta_f}}$$
(63)

$$W_t^{HI} = \left[\left(\frac{1}{1 - \gamma} \right) \int_{\gamma}^{1} [W_t^{HI}(k)]^{1 - \vartheta_f} di \right]^{\frac{1}{1 - \vartheta_f}}$$

$$\tag{64}$$

Optimal allocation of labour derives the following supply functions in each category for given level of wage income.

$$[L_t^{HF}(k)] = \left(\frac{1}{\gamma}\right) \left(\frac{[W_t^{HF}(k)]}{W_t^{HF}}\right)^{-\vartheta_f} L_t^{HF} \tag{65}$$

$$[L_t^{HI}(k)] = \left(\frac{1}{1-\gamma}\right) \left(\frac{[W_t^{HI}(k)]}{W_t^{HI}}\right)^{-\vartheta_f} L_t^{HI}$$
(66)

$$L_t^{HF} = \gamma \left(\frac{W_t^{HF}}{W_t}\right)^{-\vartheta_f} L_t \tag{67}$$

$$L_t^{HI} = (1 - \gamma) \left(\frac{W_t^{HI}}{W_t}\right)^{-\vartheta_f} L_t \tag{68}$$

Economy-wide and sector-wise wage income can be given as:

$$\int_{0}^{\gamma} [L_{t}^{HF}(k)][W_{t}^{HF}(k)]dj + \int_{\gamma}^{1} [L_{t}^{HI}(k)][W_{t}^{HI}(k)]dj = L_{t}W_{t}$$
(69)

$$\int_{0}^{\gamma} [L_{t}^{HF}(k)][W_{t}^{HF}(k)]dj = L_{t}^{HF}W_{t}^{HF}$$
(70)

$$\int_{V}^{1} [L_{t}^{HI}(k)][W_{t}^{HI}(k)]dj = L_{t}^{HI}W_{t}^{HI}$$
(71)

Where

$$L_t^{HF}W_t^{HF} + L_t^{HI}W_t^{HI} = L_tW_t (72)$$

Formulization of Lagrangian by (2) and (4)

$$\mathcal{L} = \underbrace{\underbrace{Max}_{\left\{C_{t}, L_{t}, \frac{M_{t}}{P_{t}}, B_{t}\right\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \beta^{t} \left\{ \frac{C_{t}^{1-\varepsilon}}{1-\varepsilon} + \eta \frac{G_{t}^{1-\kappa}}{1-\kappa} + \frac{\left(\frac{M_{t}}{P_{t}}\right)^{1-\tau}}{1-\tau} - \frac{L_{t}^{1+\nu}}{1+\nu} \right\} \\
- \lambda_{t} \left\{C_{t} P_{t} + M_{t} + Q_{t} B_{t} - M_{t-1} - B_{t-1} - L_{t} W_{t} - \Omega_{t} - \eta G_{t}^{T} + T_{t}^{d} \right\} \tag{73}$$

Kuhn-Tucker conditions:

$$C_t: \beta^t C_t^{-\varepsilon} - \lambda_t P_t = 0 \tag{74}$$

$$L_t: -\beta^t L_t^{\nu} + \lambda_t W_t = 0 \tag{75}$$

$$\frac{M_t}{P_t} : \beta^t \left(\frac{M_t}{P_t}\right)^{-\tau} - \lambda_t P_t + E_t \lambda_{t+1} P_t = 0 \tag{76}$$

$$B_t: -\lambda_t Q_t + E_t \lambda_{t+1} = 0 \tag{77}$$

(74) and (77) collapse to (78)

$$\frac{\beta^t C_t^{-\varepsilon}}{\beta^{t+1} E_t C_{t+1}^{-\varepsilon}} = \frac{\lambda_t P_t}{E_t \lambda_{t+1} E_t P_{t+1}}$$

$$\Rightarrow 1 = E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\varepsilon} \right\} \frac{\beta}{Q_t} \tag{78}$$

(74) and (75) yield (79)

$$\frac{\beta^t L_t^{\nu}}{\beta^t C_t^{-\varepsilon}} = \frac{\lambda_t W_t}{\lambda_t P_t}$$

$$\Rightarrow C_t^{\varepsilon} L_t^{\nu} = \frac{W_t}{P_t}$$
(79)

(74), (76) and (77) collapse to (80)
$$\frac{\beta^{t} \left(\frac{M_{t}}{P_{t}}\right)^{-\tau}}{\beta^{t} C_{t}^{-\varepsilon}} = \frac{\lambda_{t} P_{t} - E_{t} \lambda_{t+1} P_{t}}{\lambda_{t} P_{t}}$$

$$\Rightarrow \frac{M_{t}}{P_{t}} = C_{t}^{\frac{\varepsilon}{\tau}} \left(\frac{1 + i_{t}}{i_{t}}\right)^{\frac{1}{\tau}}$$
(80)
$$\operatorname{Since} Q_{t} = \frac{1}{1 + i_{t}}$$

(78) is optimal consumption-saving decision (optimal inter-temporal consumption decision), (79) is optimal consumption-leisure decision (optimal consumption-labour supply decision) and (80) is optimal consumption-demand of real balances decision of households. Households' preferences, (78), (79) and (80) do not have any influence of actual public consumption, ηG_t and of government transfers, G_t^T . Thus, public spending on consumption, government transfer, political/bureaucratic corruption/embezzlement in public spending on consumption and in government transfer do not affect optimal consumption-saving decision, optimal consumption-leisure decision and optimal consumption-demand of real balances decision of households.

4. Conclusion

In this study the Indian economy, dominated to informal sector, has been modeled in such a way to maximize integrated utility of all of its inhabited households through maximizing the utility of a representative household. Households maximize their utility by making their consumption-saving, consumption-leisure and consumption-demand of real balances decisions optimal. Again how these decisions of households are affected by public spending on consumption, by government transfer, by political/bureaucratic corruption/embezzlement in public spending on consumption and by political/bureaucratic corruption/embezzlement in government transfer. With the

help of complex algebra and calculus the paper shows households decisions of consumption-saving, consumption-leisure and consumption-demand of real balances are not affected by public spending on consumption, by government transfer, by political/bureaucratic corruption/embezzlement in public spending on consumption and by political/bureaucratic corruption/embezzlement in government transfer. This result is completely based on pure theory. The paper provides opportunities for further empirical research on the topic.

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SUMMARY OF THE PhD THESIS ENTITLED

"MACROECONOMIC GOALS AND INFLATION TARGETING IN INDIA"

SUBMITTED TO



UNIVERSITY OF KOTA, KOTA

GIRISH KUMAR PALIWAL

UNDER THE SUPERVISION OF

Dr. G. L. MALAV

LECTURER, GOVERNMENT COLLEGE, KOTA

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SUMMARY OF THE STUDY

"MACROECONOMIC GOALS AND INFLATION TARGETING IN INDIA"

It is not an easy task to find many areas in macroeconomics where almost full agreement has emerged in the last few years. However, there is today a widespread growing consensus amongst leading policy makers and academic macroeconomists that the single most important goal of monetary policy should be the pursuit of price stability, Blejer and Leone (1999). To chase the price stability central banks have recently developed a new policy tactic called inflation targeting. Inflation targeting central bank targets publically announced numerical target for annual inflation through policy instrument to stabilize inflation itself and real variables of the economy. Price stability remains prime goal of the monetary authority while other goals become subsidiary during the inflation targeting regime. To decide the monetary policy instrument many variables come into picture apart from monetary aggregates and exchange rate. To conduct inflation targeting, a high degree of transparency by publishing objectives, decisions and plans of the central bank for public is indispensable. Inflation targeting central bank has always an obligation and accountability to meet the objectives. New Zealand was the pioneer one to adopt the inflation targeting regime in 1990 and as of now i.e. May, 2013, a total of 27 industrialized and non-industrialized countries have adopted the inflation targeting regime. The inflation targeting regime has been very successful, in terms of first stabilizing the inflation and then real variables of the economy, as hitherto no country has abandoned after taking it up or even articulated any regret.

In my model optimizing private sector (households' and firms') behavior is represented by two structural equations, an aggregate supply equation (a forward looking 'New Keynesian Phillips Curve', NKPC), an aggregate demand equation ('Dynamic IS curve', DISC), and Taguchi (Monetary Policy) Loss Function. The aggregate supply equation, NKPC, is derived from a first order condition for optimal price setting by the representative supplier (firm) following Clarida, Galí, and Gertler (1999) along the lines of Calvo sticky pricing model, Calvo (1983). Even though there are more realistic formulations, Taylor (1979, 1980) and Fischer (1977)), Calvo pricing is more comfortable, simple and gives very similar results in comparison to more complicated models. An aggregate demand equation, DISC, is derived from an Euler consumption equation for the optimal timing of purchases following Woodford

(1999) along the lines of Dixit Stiglitz (1977). In the model, inflation and output are both predetermined for one period, as in Bernanke and Woodford (1997), Rotemberg and Woodford (1997, 1999), and Svensson (2003), except for an unforecastable random error term that cannot be affected by monetary policy. Taguchi Loss Function (Taguchi Method), Taguchi (1986), is used to calculate the loss caused to the society for an off target quality characteristic. Variables of the economy (inflation and output gap) are introduced to write the Taguchi Loss Function. Taguchi Loss Function is minimized, subject to NKPC and DISC to get the optimal reaction (the instrument rate) of the Reserve Bank of India to hit the inflation target.

The structure of emerging market economies is somewhat differ than that of advance economies due to existence of large informal sector. The structure of goods, labour and credit markets are pretty dissimilar in formal and informal sectors of the economy as agents have different endowments and constraints. In the advance economies the relative size of informal sector is much smaller to that of formal sector; therefore, it is reasonable to ignore the informal sector in advanced economies as it has negligible impact on the aggregates. But in the emerging market economies where the informal sector is relatively large and plays an important role in the economy then neglecting the informal sector would not be justified; Schneider et al. (2010). Informal sector plays a major role in employment generation, especially for the developing world; Agenor and Montiel (1996); Harris-White and Sinha (2007); Marjit and Kar (2011) and Dutta et al. (2011).

The Indian economy has relatively very large informal sector as the lion's share of Indian workforce works in this sector to contribute around half of its national product, NSC (2012). In such an informal economic environment this study studies the nature of domestic inflation and thereby studies the real variables of the economy i.e. output and employment. The related issues have been framed in an Open Economy New Keynesian Dynamic Stochastic General Equilibrium Model with micro-foundations to outfit the Indian economy and thereby to explain the nature of Indian domestic inflation, a key instrument for inflation targeting central banks. The Indian economy is an emerging market economy and primarily comprises of two sectors, namely, formal and informal and they are asymmetric in nature to each other. The formal sector shows sluggish prices and rigid wages and imperfections in the markets while informal sector characterizes the complete flexibility in prices and

wages and perfections in markets. Thus, Indian economy comprises of a very typical mixture of Keynesian and Classical markets. The New Keynesian Phillips Curve for Indian economy reveals that the degree of stickiness in prices in formal sector markets has a deep impact on the domestic inflation as informal sector markets are frictionless and have complete price flexibility (zero stickiness). Thus, degree of stickiness in prices in formal sector markets plays a major role to determine the domestic inflation. The study shows that when Reserve Bank of India (RBI) conducts monetary policy, the formal sector observes the fluctuations in real variables while nominal variables vary in the informal sector. In such an economic environment the study reveals that the performance of overall monetary policy is observed very poor in term of output stabilization because of this huge informal sector. Though Indian monetary authority is helpless to stabilize the real variables of the economy in the short run but at the same time it got a shiny side, the informal sector which observes only nominal effects. The study shows that RBI got a pretty good command on price level, ceteris paribus, without affecting (or negligible effect on) the output/employment. Low and stable inflation is good for economic growth and development. How to keep the inflation low and stable? Inflation targeting framework has a solution to this issue. The study shows that RBI can efficiently control the inflation through managing general price level without making any negative impact on output/employment in short run then India should adopt inflation targeting regime to keep the inflation low and stable, which in turn good for economic growth and development. The study recommends in terms of policy that if India adopts the inflation targeting regime then the negative impacts of the inflation targeting are much less than that of the positive outcomes, therefore, India should adopt inflation targeting regime. This conclusion is based on pure theory and may have deviation from reality. This study provides opportunities for further empirical work.

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